Internet Appendix for “Dividend Dynamics and the Term Structure of Dividend Strips

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Section I of this Internet Appendix reports the derivations of the long-run risk (Bansal, Kiku, and Yaron (BKY, 2012)) model. In addition, it reports the approximation errors in the baseline BKY calibration with large persistence of dividend volatility, and performs additional comparative statics analysis. Section II reports the derivations of the habit formation model (Campbell and Cochrane (CC, 1999)). Finally, Section III reports additional empirical analysis and robustness checks.

I. Long-Run Risk Model

A. Identification of \( \{\overline{z}, \kappa_0, \kappa_1, A_0, A_x, A_\sigma\} \)

BKY specify log-consumption and log-dividend dynamics as driven by two persistent variables \( (x_t, \sigma_t) \):

\[
\begin{align*}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t \tilde{\epsilon}_{c,t+1} \\
\Delta d_{t+1} &= \mu_d + \rho_d x_t + \nu_c \sigma_t \tilde{\epsilon}_{c,t+1} + \nu_d \sigma_t \tilde{\epsilon}_{d,t+1} \\
x_{t+1} &= \rho_x x_t + \nu_x \sigma_t \tilde{\epsilon}_{x,t+1} \\
\sigma^2_{t+1} &= \sigma^2 + \rho_\sigma \left( \sigma^2_t - \sigma^2 \right) + \nu_\sigma \tilde{\epsilon}_{\sigma,t+1}. \\
\end{align*}
\]

(I.A.1)

Epstein and Zin (1989) preferences imply that the log pricing kernel takes the form

\[
m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1},
\]

(I.A.2)

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where \( r_c \) is the return on the consumption claim, \( \gamma \) is relative risk aversion coefficient, \( \psi \) is the elasticity of intertemporal substitution, and \( \theta = \left( \frac{1-\gamma}{1-1/\psi} \right) \).

For tractability, two approximations are made. The first is model-specific, and assumes that the log price-dividend ratio and the log price-consumption ratio are approximately affine in the state variables:

\[
\begin{align*}
  z_t &\equiv \log \left( \frac{V_{c,t}}{C_t} \right) \approx A_0 + A_x x_t + A_{\sigma t}^2 \\
  z_{d,t} &\equiv \log \left( \frac{V_{d,t}}{D_t} \right) \approx F_0 + F_x x_t + F_{\sigma t}^2. 
\end{align*}
\] (IA.3)

The second approximation (the Campbell-Shiller (1988) approximation) is mechanical, and approximates the log return \( r_c \equiv \log R_c \) on the consumption claim to be linear in the log price-dividend ratio (a similar approximation is made for the log stock return):

\[
\begin{align*}
  r_c &= \log \left( \frac{V_{c,t+1} + C_{t+1}}{V_{c,t}} \right) \\
  &= \log \left[ \left( \frac{V_{c,t+1} + C_{t+1}}{C_{t+1}} \right) \left( \frac{C_{t+1}}{C_t} \right) \left( \frac{C_t}{V_{c,t}} \right) \right] \\
  &= \log (1 + e^{z_{t+1}}) + \Delta c_{t+1} - z_t \\
  &\approx \log (1 + e^{\bar{z}} (1 + (z_{t+1} - \bar{z}))) + \Delta c_{t+1} - z_t \\
  &\approx \log (1 + e^{\bar{z}}) + \left( \frac{e^{\bar{z}}}{1 + e^{\bar{z}}} \right) (z_{t+1} - \bar{z}) + \Delta c_{t+1} - z_t \\
  &\approx \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1}, \quad \text{(IA.4)}
\end{align*}
\]

where

\[
\begin{align*}
  \kappa_1 &= \left( \frac{e^{\bar{z}}}{1 + e^{\bar{z}}} \right) \quad \text{(IA.5)} \\
  \kappa_0 &= \log(1 + e^{\bar{z}}) - \bar{z} \left( \frac{e^{\bar{z}}}{1 + e^{\bar{z}}} \right) \\
  &= - \log(1 - \kappa_1) - \kappa_1 \log \left( \frac{\kappa_1}{1 - \kappa_1} \right). \quad \text{(IA.6)}
\end{align*}
\]

Plugging these approximations into the pricing kernel, and noting that \( (\theta - 1 - \frac{\psi}{\psi}) = -\gamma \), we find

\[
\begin{align*}
  m_{t+1} &= \theta \log \delta - \gamma \Delta c_{t+1} + (\theta - 1) \kappa_0 + (\theta - 1) \kappa_1 \left( A_0 + A_x x_{t+1} + A_{\sigma t+1}^2 \right) - (\theta - 1) z_t \\
  &= \theta \log \delta + (\theta - 1) \kappa_0 - \gamma (\mu_c + x_t + \sigma_t \tilde{e}_{c,t+1}) + (\theta - 1) \kappa_1 A_0 \\
  &\quad + (\theta - 1) \kappa_1 A_x \left[ \rho_x x_t + \nu_x \sigma_t \tilde{e}_{x,t+1} \right] + (\theta - 1) \kappa_1 A_{\sigma} \left[ \sigma_{\sigma}^2 + \rho_{\sigma} \left( \sigma_{\sigma}^2 - \sigma_t^2 \right) + \nu_{\sigma} \tilde{e}_{\sigma,t+1} \right] \\
  &\quad - (\theta - 1) z_t. \quad \text{(IA.7)}
\end{align*}
\]
Hence,
\[ E_t \left[ m_{t+1} \right] = \theta \log \delta + (\theta - 1) \kappa_0 - \gamma (\mu_x + x_t) + (\theta - 1) \kappa_1 A_x \rho x_t + (\theta - 1) \kappa_1 \sigma_x \left( \sigma_t^2 + \rho \sigma_t (\sigma_t^2 - \sigma_x^2) \right) - (\theta - 1) z_t \]  
(IA.8)

and
\[ m_{t+1} - E_t \left[ m_{t+1} \right] = -\lambda_c \sigma_t \tilde{e}_{c,t+1}^c - \lambda_x \sigma_t \tilde{e}_{x,t+1}^x - \lambda_a \nu_a \tilde{e}_{a,t+1}. \]  
(IA.9)

where
\[
\begin{align*}
\lambda_c &\equiv \gamma \\
\lambda_x &\equiv (1 - \theta) \kappa_1 A_x \nu_x \\
\lambda_a &\equiv (1 - \theta) \kappa_1 A_a.
\end{align*}
\]  
(IA.10)

Hence, we have
\[ \text{Var}_t \left[ m_{t+1} \right] = \lambda_c^2 \sigma_t^2 + \lambda_x^2 \sigma_t^2 + \lambda_a^2 \nu_a^2. \]  
(IA.11)

To solve the model, we need to identify the six parameters \( \{ \tilde{z}, \kappa_0, \kappa_1, A_0, A_x, A_a \} \).

We already have three equations:
\[
\begin{align*}
\kappa_1 &= \left( \frac{e^{\tilde{z}}}{1 + e^{\tilde{z}}} \right) \quad \text{(IA.12)} \\
\kappa_0 &= \log(1 + e^{\tilde{z}}) - \tilde{z} \left( \frac{e^{\tilde{z}}}{1 + e^{\tilde{z}}} \right) \\
\tilde{z} &= A_0 + A_a \sigma_t.
\end{align*}
\]  
(IA.13)

The other three equations come from
\[ 1 = E_t \left[ e^{m_{t+1} + r_{c,t+1}} \right]. \]  
(IA.14)

Since \( m_{t+1} + r_{c,t+1} \) is normally distributed, this is equivalent to
\[ 0 = E_t \left[ m_{t+1} + r_{c,t+1} \right] + \frac{1}{2} \text{Var}_t \left[ m_{t+1} + r_{c,t+1} \right]. \]  
(IA.15)

Note that
\[
\begin{align*}
( m_{t+1} + r_{c,t+1} ) &= \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + \theta r_{c,t+1} \\
&= \theta \log \delta + \theta \kappa_0 + (1 - \gamma) (\mu_x + x_t + \sigma_t \tilde{e}_{c,t+1}^c) \\
&\quad + \theta \kappa_1 A_x + \theta \kappa_1 A_x \left( \rho_x x_t + \nu_x \sigma_t \tilde{e}_{x,t+1} \right) \\
&\quad + \theta \kappa_1 A_a \left[ \sigma_t^2 + \rho_a (\sigma_t^2 - \sigma_x^2) + \nu_a \tilde{e}_{a,t+1} \right] - \theta z_t.
\end{align*}
\]  
(IA.16)
Hence,
\[
E_t [m_{t+1} + r_{c,t+1}] = \theta \log \delta + \theta \kappa_0 + (1 - \gamma) (\mu_c + x_t) + \theta \kappa_1 A_o + \theta \kappa_1 A_x \rho_s x_t + \theta \kappa_1 A_x \left[ \sigma^2 + \rho_s (\sigma^2 - \bar{\sigma}^2) \right] - \theta z_t. \tag{IA.17}
\]

Also,
\[
(m_{t+1} + r_{c,t+1}) - E_t [m_{t+1} + r_{c,t+1}] = (1 - \gamma) \sigma_t \tilde{\varepsilon}_{c,t+1} + \theta \kappa_1 A_x \nu_x \sigma_t \tilde{\varepsilon}_{x,t+1} + \theta \kappa_1 A_x \sigma_t \nu_x \tilde{\varepsilon}_{x,t+1}. \tag{IA.18}
\]

Hence,
\[
\text{Var}_t [m_{t+1} + r_{c,t+1}] = \sigma_t^2 (1 - \gamma)^2 + \sigma_t^2 (\theta \kappa_1 A_x \nu_x)^2 + (\theta \kappa_1 A_x \nu_x)^2. \tag{IA.19}
\]

Plugging equations (IA.17) and (IA.19) into equation (IA.15) and collecting terms linear in \(x_t, \sigma_t^2\), and independent of those two state variables, we obtain the three conditions

\[
\begin{align*}
x_t : & \quad 0 = (1 - \gamma) + \theta \kappa_1 A_x \rho_s - \theta A_x \\
\sigma_t^2 : & \quad 0 = \theta \kappa_1 A_x \rho_s - \theta A_x + \frac{1}{2} (1 - \gamma)^2 + \frac{1}{2} (\theta \kappa_1 A_x \nu_x)^2 \\
\text{indep} : & \quad 0 = \theta \log \delta + \theta \kappa_0 + (1 - \gamma) \mu_c + \theta \kappa_1 A_o + \theta \kappa_1 A_x \sigma^2 (1 - \rho_s) - \theta A_o + \frac{1}{2} (\theta \kappa_1 A_x \nu_x)^2.
\end{align*}
\]

Solving, we find

\[
\begin{align*}
A_x & = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho_s} \tag{IA.20} \\
A_x & = \frac{(1 - \gamma) (1 - \frac{1}{\psi})}{2(1 - \kappa_1 \rho_s)} \left[ 1 + \frac{\kappa_2 \nu_x^2}{(1 - \kappa_1 \rho_s)^2} \right] \\
A_o & = \frac{1}{(1 - \kappa_1)} \left[ \log \delta + \kappa_0 + \left(1 - \frac{1}{\psi} \right) \mu_c + \kappa_1 A_o \sigma^2 (1 - \rho_s) + \frac{\theta}{2} (\kappa_1 A_x \nu_x)^2 \right].
\end{align*}
\]

To solve, we follow BKY by guessing a value for \(\bar{z}\), and then plugging this in to get \(\kappa_0\) and \(\kappa_1\), and in turn \(\{A_o, A_x, A_x\}\). We then identify a new guess for \(\bar{z}\) using
\[
\bar{z} = A_o + A_x \sigma^2. \tag{IA.21}
\]

We iterate until convergence. Once a fixed point has been found, the pricing kernel is identified, implying we can go forward to price the dividend claim.
B. Risk-Free Rate

The risk-free rate at date $t$ is determined from the pricing kernel via
\[ R^{-1}_f(x_t, \sigma_t) \equiv e^{-r_f(x_t, \sigma_t)} \]
\[ = E_t[e^{m_{t+1}}] \]
\[ = e^{-r_0 - \psi x_t - r_\sigma \sigma_t^2}, \tag{IA.22} \]
where we have defined
\[ -r_0 \equiv \theta \log \delta + (\theta - 1) \kappa_0 - \gamma \mu_c + (\theta - 1) \kappa_1 A_o \]
\[ + (\theta - 1) \kappa_1 A_x \sigma^2 (1 - \rho_\sigma) - (\theta - 1) A_\sigma + \frac{1}{2} \lambda_\sigma^2 \nu_\sigma^2 \]
\[ -r_\sigma \equiv \frac{1}{2} \lambda_c^2 + \frac{1}{2} \lambda_x^2 - (\theta - 1) A_\sigma (1 - \kappa_1 \rho_\sigma). \tag{IA.23} \]

C. The Dividend Claim

As noted previously, dividend dynamics are specified as
\[ \Delta d_{t+1} = \mu_d + \rho_d x_t + \nu_c \sigma_t \tilde{e}_{c,t+1} + \nu_d \sigma_t \tilde{e}_{d,t+1}. \tag{IA.24} \]
We assume that the log price-dividend ratio is approximately
\[ z_{d,t} \equiv \log \left( \frac{V_{d,t}}{D_t} \right) \approx F_0 + F_x x_t + F_\sigma \sigma_t^2. \tag{IA.25} \]
We also approximate the log stock return via
\[ r_d \approx \kappa_{od} + \kappa_{1d} z_{d,t+1} - z_{d,t} + \Delta d_{t+1}, \tag{IA.26} \]
where
\[ \kappa_{1d} = \left( \frac{e^\bar{z}_d}{1 + e^\bar{z}_d} \right) \tag{IA.27} \]
\[ \kappa_{od} = \log(1 + e^\bar{z}_d) - \bar{z}_d \left( \frac{e^\bar{z}_d}{1 + e^\bar{z}_d} \right) \]
\[ = -\log(1 - \kappa_{1d}) - \kappa_{1d} \log \left( \frac{\kappa_{1d}}{1 - \kappa_{1d}} \right). \tag{IA.28} \]
Because the parameters \{\bar{z}, \kappa_0, \kappa_1, A_0, A_x, A_\sigma\} have already been identified, here all we need to identify is \{\bar{z}_d, \kappa_{od}, \kappa_{1d}, F_0, F_x, F_\sigma\}. Again, we already have three equations: the definitions of \{\kappa_{od}, \kappa_{1d}\} and the self-consistent condition
\[ \bar{z}_d = F_0 + F_\sigma \bar{\sigma}^2. \tag{IA.29} \]
The last three conditions are determined from the following first-order condition:

\[ 0 = E_t [m_{t+1} + r_{d,t+1}] + \frac{1}{2} \text{Var}_t [m_{t+1} + r_{d,t+1}] . \]  \hspace{1cm} (IA.30)

Note that

\[
m_{t+1} + r_{d,t+1} = \theta \log \delta + (\theta - 1) \kappa_0 + \kappa_{0d} - \gamma \left( \mu_c + x_t + \sigma_t \bar{\epsilon}_{c,t+1} \right) + (\theta - 1) \kappa_1 A_0 \\
+ \kappa_{id} F_0 + [(\theta - 1) \kappa_1 A_x + \kappa_{id} F_x] \left[ \rho_x x_t + \nu_x \sigma_t \bar{\epsilon}_{x,t+1} \right] \\
+ [(\theta - 1) \kappa_1 A_\sigma + \kappa_{id} F_\sigma] \left[ \sigma^2 + \rho_\sigma \left( \sigma_t^2 - \bar{\sigma}^2 \right) \right] \\
- (\theta - 1) \delta_t - z_{d,t} + \mu_d + \rho_d x_t + \nu_c \sigma_t \bar{\epsilon}_{c,t+1} + \nu_d \sigma_t \bar{\epsilon}_{d,t+1}. \hspace{1cm} (IA.31)
\]

Hence,

\[
E_t [m_{t+1} + r_{d,t+1}] = \theta \log \delta + (\theta - 1) \kappa_0 - \gamma \left( \mu_c + x_t \right) + (\theta - 1) \kappa_1 A_0 \\
+ [(\theta - 1) \kappa_1 A_x + \kappa_{id} F_x] \rho_x x_t \\
+ [(\theta - 1) \kappa_1 A_\sigma + \kappa_{id} F_\sigma] \left[ \sigma^2 + \rho_\sigma \left( \sigma_t^2 - \bar{\sigma}^2 \right) \right] \\
- (\theta - 1) \delta_t - z_{d,t} + \mu_d + \rho_d x_t. \hspace{1cm} (IA.32)
\]

Also,

\[
\left\{ (m_{t+1} + r_{d,t+1}) - E_t [m_{t+1} + r_{d,t+1}] \right\} = - \gamma \sigma_t \bar{\epsilon}_{c,t+1} \\
+ [(\theta - 1) \kappa_1 A_x + \kappa_{id} F_x] \nu_x \sigma_t \bar{\epsilon}_{x,t+1} \\
+ [(\theta - 1) \kappa_1 A_\sigma + \kappa_{id} F_\sigma] \nu_\sigma \bar{\epsilon}_{\sigma,t+1} \\
+ \nu_c \sigma_t \bar{\epsilon}_{c,t+1} + \nu_d \sigma_t \bar{\epsilon}_{d,t+1}. \hspace{1cm} (IA.33)
\]

This implies

\[
\text{Var}_t [m_{t+1} + r_{d,t+1}] = \sigma^2_c (\nu_c - \gamma)^2 + \nu_d^2 \sigma^2_t + [(\theta - 1) \kappa_1 A_x + \kappa_{id} F_x]^2 \nu^2_x \\
+ \sigma^2_\sigma^2 \left[ (\theta - 1) \kappa_1 A_\sigma + \kappa_{id} F_\sigma \right]^2. \hspace{1cm} (IA.34)
\]

Plugging equations (IA.32) and (IA.34) into equation (IA.30) and collecting terms
Further, we define the log expected excess return as

\[ x_1 : 0 = -\gamma + (\theta - 1)\kappa_x A_x\rho_x + \kappa_{1d}F_x\rho_x - (\theta - 1)A_x - F_x + \rho_d \]  
(IA.35)

\[ \sigma_t^2 : 0 = \left[ (\theta - 1)\kappa_x A_x + \kappa_{1d}F_x \right] \rho_x - (\theta - 1)A_x - F_x + \frac{1}{2} (\nu_c - \gamma)^2 \]
\[ + \frac{1}{2} \nu_x^2 \left[ (\theta - 1)\kappa_x A_x + \kappa_{1d}F_x \right]^2 + \frac{1}{2} \nu_d^2 \]
const : 0 = \theta \log \delta + (\theta - 1)\kappa_0 - \gamma \mu_c + (\theta - 1)\kappa_1 A_0
\[ + \left[ (\theta - 1)\kappa_x A_x + \kappa_{1d}F_x \right] \sigma_x^2 (1 - \rho_x) - (\theta - 1)A_0 + \kappa_{od} \]
\[ + \kappa_{1d}F_0 - F_0 + \mu_d + \frac{\nu_x^2}{2} \left[ (\theta - 1)\kappa_x A_x + \kappa_{1d}F_x \right]^2. \]

Simplifying, this gives

\[ F_x = \frac{\rho_d - \frac{1}{\psi}}{1 - \kappa_{1d}\rho_x} \]  
(IA.36)

\[ F_\sigma = \left( \frac{1}{1 - \kappa_{1d}\rho_x} \right) \left[ (\theta - 1) (\kappa_x\rho_x - 1) A_x + \frac{1}{2} [(\theta - 1)\kappa_x A_x + \kappa_{1d}F_x]^2 \nu_x^2 \right. 
\[ + \frac{1}{2} (\nu_c - \gamma)^2 + \frac{\nu_d^2}{2} \] \left( IA.37 \right)

\[ F_0 = \left( \frac{1}{1 - \kappa_{1d}} \right) \left[ \log \delta - \frac{1}{\psi} \mu_c - (\theta - 1)\frac{1}{2} (\kappa_x A_x \nu_x)^2 + \kappa_{1d}F_\sigma \sigma_x^2 (1 - \rho_x) \right. \]
\[ + \kappa_{od} + \mu_d + \frac{\nu_x^2}{2} (\kappa_{1d}F_\sigma - \lambda_\sigma)^2 \] \left( IA.38 \right).

These results correct some typos in the appendix of BKY.

Combining equations (IA.22) and (IA.26), we express the log excess stock return as

\[ (r_{d,t+1} - r_{f,t}) = \kappa_{od} + \kappa_{1d} z_{d,t+1} - z_{d,t} + \Delta d_{t+1} - r_{f,t}. \]  
(IA.39)

Further, we define the log expected excess return as

\[ \bar{\tau}_{d,t} = \log E_t \left[ e^{(r_{d,t+1} - r_{f,t})} \right] \]
\[ = \kappa_{od} + \kappa_{1d}F_0 + \kappa_{1d}F_x \rho_x x_t + \kappa_{1d}F_\sigma \left[ \sigma_x^2 (1 - \rho_x) + \rho_x \sigma_t^2 \right] - F_0 - F_x x_t - F_\sigma^2 + \mu_d \]
\[ + \rho_d x_t - r_o - \frac{1}{\psi} x_t - r_\sigma \sigma_t^2 \]
\[ + \frac{1}{2} \left( \kappa_{1d} F_\sigma \nu_x \right)^2 \sigma_t^2 + \frac{1}{2} \left( \kappa_{1d} F_\sigma \nu_x \right)^2 + \frac{1}{2} \sigma_t^2 (\nu_c^2 + \nu_d^2). \]  
(IA.40)

This simplifies to

\[ \bar{\tau}_{d,t} = \nu_x^2 \kappa_{1d} F_\sigma \lambda_\sigma + \left( \nu_x \kappa_{1d} F_\sigma \lambda_x + \nu_c \lambda_\sigma \right) \sigma_t^2. \]  
(IA.41)
We also define the stock volatility as

\[
\sigma_{d,t} = \sqrt{\log E_t \left[ e^{2(r_{d,t+1} - r_{f,t})} \right] - \log \left( E_t \left[ e^{(r_{d,t+1} - r_{f,t})} \right] \right)^2} = \sqrt{(\kappa_d F_x \nu_x)^2 + \sigma_t^2 \left[ \nu_x^2 + \nu_d^2 + (\kappa_d F_x \nu_x)^2 \right]}.
\]

To get the term structure of dividends, define

\[
G_d^{(j,T)}(t, d_t, x_t, \sigma_t) = E_t \left[ e^{j T} \right],
\]

\[
G_d^{(j,T)}(t-1, d_{t-1}, x_{t-1}, \sigma_{t-1}) = E_{t-1} \left[ e^{j T} \right].
\]

Because the state vector dynamics are affine, it is well known (e.g., Duffie and Kan (1996)) that the solution is of the form

\[
G_d^{(j,T)}(t, d_t, x_t, \sigma_t) = e^{j d_t + F_{0,j}(n) + F_{x,j}(n) x_t + F_{\sigma,j}(n) \sigma_t^2} \]

\[
G_d^{(j,T)}(t-1, d_{t-1}, x_{t-1}, \sigma_{t-1}) = e^{j d_{t-1} + F_{0,j}(n+1) + F_{x,j}(n+1) x_{t-1} + F_{\sigma,j}(n+1) \sigma_{t-1}^2},
\]

where we have defined \( n = (T-t) \) as the number of periods between dates \( t \) and \( T \).

Note that for the solution to be self-consistent at \( t = T \), we must have the boundary conditions \( F_{0,j}(0) = 0 \), \( F_{x,j}(0) = 0 \), and \( F_{\sigma,j}(0) = 0 \).

By the law of iterated expectations, we have

\[
G_d^{(j,T)}(t-1, d_{t-1}, x_{t-1}, \sigma_{t-1}) = E_{t-1} \left[ E_t \left[ e^{j T} \right] \right] = E_{t-1} \left[ G_d^{(j,T)}(t, d_t, x_t, \sigma_t) \right].
\]

Plugging equations (IA.44) and (IA.45) into equation (IA.46), performing the expectation, and then collecting terms linear in \( x_t, \sigma_t^2 \) and independent of the state vector, we obtain the three recursive equations

\[
F_{x,j}(n+1) = \rho_x F_{x,j}(n) + j \rho_d
\]

\[
F_{0,j}(n+1) = F_{0,j}(n) + F_{\sigma,j}(n) \bar{\sigma}^2 (1 - \rho_x) + j \mu_d + \frac{\nu_x^2}{2} F_{\sigma,j}(n)
\]

\[
F_{\sigma,j}(n+1) = \rho_x F_{\sigma,j}(n) + \frac{1}{2} \nu_x^2 F_{x,j}(n) + \frac{1}{2} j^2 \left( \nu_x^2 + \nu_d^2 \right).
\]

Setting \( j = 1 \), we can determine the term structure of dividend expected growth rates over horizon \( n \) via

\[
g_{d,n} = \frac{1}{n} \log \left( E_t \left[ e^{d_T - d_t} \right] \right) = \left( \frac{1}{n} \right) \left[ F_{0,1}(n) + F_{x,1}(n) x_t + F_{\sigma,1}(n) \sigma_t^2 \right].
\]
Similarly, using both $j = 2$ and $j = 1$, we define the term structure of dividend volatilities via

$$
\sigma_{d,n} = \sqrt{\left(\frac{1}{n}\right) \left[ \log \mathbb{E}_t \left[ e^{2(d_T - d_t)} \right] - \log \left( \mathbb{E}_t \left[ e^{d_T - d_t} \right] \right)^2 \right]}
$$

(IA.49)

$$
= \left\{ \left(\frac{1}{n}\right) \left[ (F_{0,2}(n) - 2F_{0,1}(n)) + (F_{x,2}(n) - 2F_{x,1}(n)) x_t 
+ (F_{x,2}(n) - 2F_{x,1}(n)) \sigma_t^2 \right] \right\}^{\frac{1}{2}}
$$

$$
= \left\{ \left(\frac{1}{n}\right) \left[ (F_{0,2}(n) - 2F_{0,1}(n)) + (F_{x,2}(n) - 2F_{x,1}(n)) \sigma_t^2 \right] \right\}^{\frac{1}{2}},
$$

where the last step follows because

$$
F_{x,2}(n) = 2F_{x,1}(n)
$$

$$
= \left( \frac{2\rho_d}{1 - \rho_x} \right) [1 - \rho_x^n].
$$

(IA.50)

### D. Dividend Strips

The date $t$ claim to the dividend $e^{d_T}$ paid out at date $T$ is defined as

$$
V_d^T(t, d_t, x_t, \sigma_t) = \mathbb{E}_t \left[ e^{(\sum_{i=1}^n m_{t+i}) + d_T} \right].
$$

(IA.51)

Note that from the law of iterated expectations we have

$$
V_d^T(t - 1, d_{t-1}, x_{t-1}, \sigma_{t-1}) = \mathbb{E}_{t-1} \left[ e^{(\sum_{i=0}^n m_{t+i}) + d_T} \right]
$$

$$
= \mathbb{E}_{t-1} \left[ \mathbb{E}_t \left[ e^{m_t + (\sum_{i=1}^n m_{t+i}) + d_T} \right] \right]
$$

$$
= \mathbb{E}_{t-1} \left[ e^{m_t} V_d^T(t, d_t, x_t, \sigma_t) \right].
$$

(IA.52)

Again, since the state vector dynamics are affine, the solution is of the form

$$
V_d^T(t, d_t, x_t, \sigma_t) = e^{d_t + F_0(n) + F_x(n)x_t + F_\sigma(n)\sigma_t^2}
$$

(IA.53)

$$
V_d^T(t - 1, d_{t-1}, x_{t-1}, \sigma_{t-1}) = e^{d_{t-1} + F_0(n+1) + F_x(n+1)x_{t-1} + F_\sigma(n+1)\sigma_{t-1}^2}.
$$

(IA.54)

The final conditions are $F_0(0) = 0$, $F_x(0) = 0$, and $F_\sigma(0) = 0$.

Plugging equations (IA.53) and (IA.54) into equation (IA.52), performing the expectation, and then collecting terms independent of the state vector and linear in $(x_t, \sigma_t^2)$,
we obtain the three recursive equations

\begin{align*}
x_t : & \quad F_x(n + 1) = -\gamma + (\theta - 1)\kappa_x A_x \rho_x - (\theta - 1)A_x + \rho_d + F_x(n)\rho_x \\
\sigma_t^2 : & \quad F_\sigma(n + 1) = (\theta - 1)\kappa_\sigma A_\sigma \rho_\sigma - (\theta - 1)A_\sigma + F_\sigma(n)\rho_\sigma + \frac{1}{2}(\nu_\sigma - \lambda_\sigma)^2 \\
\text{indep} : & \quad F_\nu(n + 1) = \theta \log \delta + (\theta - 1)\kappa_\nu - \gamma \mu_\nu + (\theta - 1)\kappa_\nu A_\nu \\
& \quad + (\theta - 1)\kappa_\nu A_\nu (1 - \rho_\sigma) - (\theta - 1)A_\nu + \mu_d \\
& \quad + F_\nu(n) + F_\nu(n)\sigma_t^2 (1 - \rho_\sigma) + \frac{\nu_\sigma^2}{2} (F_\nu(n) - \lambda_\sigma)^2.
\end{align*}

To obtain the dividend strip returns, define the date \( t \) one-period gross return on a strip that matures at date \( T \) via

\[
\tilde{R}_{t+1}^T \equiv e^{\tilde{r}_{t+1}^T} = \frac{V_T(t + 1, x_{t+1}, \sigma_{t+1})}{V_d(t, x_t, \sigma_t)}.
\]

The log expected excess returns for the dividend strips are

\[
\bar{\tau}(V_d^T(t)) = \log E_t \left[ e^{(\tilde{r}_{t+1}^T - r_f, t)} \right]
\]

\[
= \mu_d + \rho_d x_t + F_\nu(n - 1) + F_\sigma(n - 1)\rho_x x_t + F_\sigma(n - 1) \left[ \sigma_t^2 (1 - \rho_\sigma) + \rho_\sigma \sigma_t^2 \right] - F_\nu(n) - F_\sigma(n) x_t - F_\sigma(n) \sigma_t^2 - r_0 - \frac{1}{\psi_\nu} x_t - r_\sigma \sigma_t^2 \\
+ \frac{1}{2} (F_\sigma(n - 1) \nu_\sigma^2) \sigma_t^2 + \frac{1}{2} (F_\nu(n - 1) \nu_\sigma^2)(\nu_\sigma^2 + \nu_\nu^2).
\]

This simplifies to

\[
\bar{\tau}(V_d^T(t)) = \nu_\sigma^2 F_\sigma(n - 1) \lambda_\sigma + (\nu_\nu \lambda_\sigma + \nu_\sigma F_\sigma(n - 1) \lambda_\sigma) \sigma_t^2.
\]

We also define dividend strip volatility as

\[
\sigma(V_d^T(t)) = \sqrt{\log E_t \left[ e^{2(\tilde{r}_{t+1}^T - r_f, t)} \right] - \log \left( E_t \left[ e^{(\tilde{r}_{t+1}^T - r_f, t)} \right] \right)^2}
\]

\[
= \sqrt{(F_\sigma(n - 1) \nu_\sigma^2) \sigma_t^2 + (F_\nu(n - 1) \nu_\sigma^2) \sigma_t^2 + \nu_\nu^2 (\nu_\sigma^2 + \nu_\nu^2)}.
\]

E. Modified BKY Model

Since the modified BKY model for EBIT is structurally equivalent to the BKY model for dividends, we omit replicating the same proofs and instead jump to the new results.
E.1. Leverage Dynamics

The claim to EBIT is approximated as

\[ V_y(y_t, x_t, \sigma_t) \approx e^{y_t + U_0 + U_x x_t + U_\sigma \sigma_t^2}. \] (IA.59)

Here, we assume that at all dates \( t \) the firm issues riskless debt that matures at date \( (t + dt) \) with present value equal to

\[ B(\ell_t, y_t, x_t, \sigma_t) = e^{\ell_t + y_t + U_0 + U_x x_t + U_\sigma \sigma_t^2} \approx e^{\ell_t} V_y(y_t, x_t, \sigma_t). \] (IA.60)

We interpret \( e^{\ell_t} \approx \frac{B(\ell_t, y_t, x_t, \sigma_t)}{V_y(y_t, x_t, \sigma_t)} \) as the leverage of the firm. Since it is riskless, the firm must pay \( e^{r_f(x_t, \sigma_t)} B(\ell_t, y_t, x_t, \sigma_t) \) at date \( (t + 1) \). It does so by issuing at this time debt with face value \( B(\ell_{t+1}, y_{t+1}, x_{t+1}, \sigma_{t+1}) \), with all residual cash flows paid out as dividends. As such, dividends \( D(t + 1) \) paid out at date \( (t + 1) \) are

\[ D(t + 1) = B(\ell_{t+1}, y_{t+1}, x_{t+1}, \sigma_{t+1}) - e^{r_f(x_t, \sigma_t)} B(\ell_t, y_t, x_t, \sigma_t) + e^{y_{t+1}}. \] (IA.61)

We specify the dynamics of log-leverage as

\[ \ell_{t+1} = 7 + \rho_\ell (\ell_t - 7) + \rho_{\ell c} x_t + \rho_{\ell \sigma}(\sigma_t^2 - \sigma^2) - \nu_{yc} \sigma_t \tilde{c}_{c,t+1} - \nu_y \sigma_t \tilde{y}_{y,t+1} - U_x \nu_x \sigma_t \tilde{c}_{x,t+1} - U_\sigma \nu_\sigma \tilde{\sigma}_{t+1}. \] (IA.62)

Hence, dividends follow the endogenously determined process

\[ D(t + 1) = e^{y_t} \left[ e^{7 + \rho_\ell (\ell_t - 7) + \rho_{\ell c} x_t + \rho_{\ell \sigma}(\sigma_t^2 - \sigma^2) + \nu_{yc} \sigma_t \tilde{c}_{c,t+1} + \nu_y \sigma_t \tilde{y}_{y,t+1} - U_x \nu_x \sigma_t \tilde{c}_{x,t+1} - U_\sigma \nu_\sigma \tilde{\sigma}_{t+1}} - e^{\ell_t + U_0 + U_x x_t + U_\sigma \sigma_t^2 + r_0 + \frac{1}{r} x_t + r_\sigma \sigma_t^2} \right] + e^{(\nu_y + \nu_\sigma \sigma_t + \nu_{yc} \sigma_t \tilde{c}_{c,t+1} + \nu_y \sigma_t \tilde{y}_{y,t+1})} \]

\[ \equiv D_1(t + 1) - D_2(t + 1) + e^{y_{t+1}}. \] (IA.63)

E.2. Dividend Strips

Here we provide a closed-form expression for the price of dividend strips, defined as

\[ V_d(t, \ell_t, y_t, x_t, \sigma_t) = E_t \left[ e^{(\sum_{i=1}^{n} m_{t+i}) D(T)} \right] = E_t \left[ e^{(\sum_{i=1}^{n} m_{t+i}) (D_1(T) - D_2(T) + e^{y_T})} \right]. \] (IA.65)

The price of dividend strips is a sum of three terms, each of which can be expressed in an exponential-affine form. The first term is

\[ V_d(t, \ell_t, y_t, x_t, \sigma_t) = E_t \left[ e^{(\sum_{i=1}^{n} m_{t+i}) B(\ell_T, y_T, x_T, \sigma_T)} \right]. \] (IA.66)
The second term is

\[ V^T_{d,2}(t, \ell_t, y_t, x_t, \sigma_t) = E_t \left[ e^{\sum_{i=0}^{n} m_t + i} B(\ell_{T-1}, y_{T-1}, x_{T-1}, \sigma_{T-1}) e^{r_f(x_{T-1}, \sigma_{T-1})} \right] \]

\[ = E_t \left[ e^{\sum_{i=1}^{n} m_t + i} B(\ell_{T-1}, y_{T-1}, x_{T-1}, \sigma_{T-1}) E_{T-1} \left[ e^{m_T + r_f(x_{T-1}, \sigma_{T-1})} \right] \right] \]

\[ = E_t \left[ e^{(\sum_{i=1}^{n} m_t + i)} B(\ell_{T-1}, y_{T-1}, x_{T-1}, \sigma_{T-1}) \right] = V^T_{d,1}(t, \ell_t, y_t, x_t, \sigma_t). \] (IA.67)

The third term is the claim to the EBIT strip, \( V^T_y(t, y_t, x_t, \sigma_t) \).

From the law of iterated expectations, the first term satisfies

\[ V^T_{d,1}(t-1, \ell_{t-1}, y_{t-1}, x_{t-1}, \sigma_{t-1}) = E_{t-1} \left[ e^{(\sum_{i=0}^{n} m_{t-1} + i)} B(\ell_{T-1}, y_{T-1}, x_{T-1}, \sigma_{T-1}) \right] \]

\[ = E_{t-1} \left[ E_t \left[ e^{m_t + (\sum_{i=1}^{n} m_{t-1} + i)} B(\ell_t, y_t, x_t, \sigma_t) \right] \right] \]

\[ = E_{t-1} \left[ e^{m_t V^T_{d,1}(t, \ell_t, y_t, x_t, \sigma_t)} \right]. \] (IA.68)

Because the state vector dynamics are affine, we know the solution is of the form

\[ V^T_{d,1}(t, \ell_t, y_t, x_t, \sigma_t) = e^{y_t + F_0(n) + F_{\ell} n + F_x(n) x_t + F_{\sigma} n \sigma_t^2} \] (IA.69)

\[ V^T_{d,1}(t-1, \ell_{t-1}, y_{t-1}, x_{t-1}, \sigma_{t-1}) = e^{y_{t-1} + F_0(n+1) + F_{\ell} (n+1) \ell_{t-1} + F_x(n+1) x_{t-1} + F_{\sigma} (n+1) \sigma_{t-1}^2}. \] (IA.70)

The final conditions are \( F_0(0) = U_0 \), \( F_{\ell}(0) = 1 \), and \( F_x(0) = U_x \), \( F_{\sigma}(0) = U_{\sigma} \).

Plugging equations (IA.69) and (IA.70) into equation (IA.68), performing the expectation, and then collecting terms independent of the state vector and linear in \((\ell_t, x_t, \sigma_t^2)\),
we obtain the four recursive equations

\[ \ell_t : \quad F_\ell(n + 1) = \rho_\ell F_\ell(n) \]
\[ x_t : \quad F_x(n + 1) = -\gamma + (\theta - 1)\kappa_\ell A_x + (\theta - 1)A_x + \rho_y + F_y(n)\rho_x + F_x(n)\rho_x + F_\ell(n)\rho_x + F_x(n)\rho_x + F_\ell(n)\rho_x + F_x(n)\rho_x + F_\ell(n)\rho_x + F_x(n)\rho_x \]
\[ \sigma_t^2 : \quad F_\sigma(n + 1) = (\theta - 1)\kappa_x A_x + (\theta - 1)A_x + F_x(n)\rho_x + F_\ell(n)\rho_x + F_x(n)\rho_x + F_\ell(n)\rho_x + F_x(n)\rho_x + F_\ell(n)\rho_x + F_x(n)\rho_x + F_\ell(n)\rho_x + F_x(n)\rho_x \]
\[ \text{indep} : \quad F_0(n + 1) = \theta \log \delta + (\theta - 1)\kappa_0 A_x + (\theta - 1)A_x + \mu_y + F_y(n)\rho_x + F_\ell(n)\rho_x + F_x(n)\rho_x + F_\ell(n)\rho_x + F_x(n)\rho_x + F_\ell(n)\rho_x + F_x(n)\rho_x + F_\ell(n)\rho_x + F_x(n)\rho_x + F_\ell(n)\rho_x + F_x(n)\rho_x + F_\ell(n)\rho_x + F_x(n)\rho_x \]

To obtain the dividend strip returns, we define the gross excess return on dividend strips as

\[ \begin{align*}
\frac{\bar{R}_{d,t+1}}{R_{d,t}(t)} &= \frac{V_{d,t+1}^T}{V_{d,t}^T R_{d,t}(t)} \\
&= \left( \frac{1}{V_{d,t}^T R_{d,t}(t)} \right) \left[ V_{d1}^T(t + 1) - V_{d1}^T(t + 1) + V_{d1}^T(t + 1) \right] \quad (IA.71)
\end{align*} \]

The expectation has three terms:

\[ \begin{align*}
\mathbb{E}_t \left[ V_{d1}^T(t + 1) \right] &= e^{\nu_1 + \mu_y + \rho_x x_t + F_y(n-1) + F_{\ell}(n-1)} \left[ \nu_2 + \rho_x (x_t - \bar{x}) + \rho_{\ell x} x_t + \rho_{\ell \sigma} (\sigma_t^2 - \bar{\sigma}^2) \right] \\
&\times e^{F_x(n-1) \rho_x x_t + F_\sigma(n-1) \left[ \bar{\sigma}^2 + \rho_\sigma (\sigma_t^2 - \bar{\sigma}^2) \right]} \\
&\times e^{\frac{\nu_2^2}{2}} \left[ \nu_2^2 + \nu_y^2 \right] \left[ 1 - F_{\ell}(n-1) \right]^2 + \left( \frac{\nu_2^2}{2} \right) \nu_y^2 \left[ F_x(n-1) - U_x F_x(n-1) \right]^2 \\
&\times e^{\frac{\nu_2^2}{2}} \left[ F_x(n-1) - U_x F_x(n-1) \right]^2
\end{align*} \]
The second moments are

\[
E_t \left[ V^{T}_{d2} (t + 1) \right] = e^{yt + \mu_y + \rho_y x_t + F_0 (n - 2) + F_1 (n - 2) \left[ \tau + \nu \right] + \rho_{x_t} x_t + \rho_{\sigma^2} (\sigma^2 - \sigma^2) } \\
\times e^{F_x (n - 2) \rho_x x_t + F_\sigma (n - 2) \left[ \sigma^2 + \rho_\sigma \left( \sigma^2 - \sigma^2 \right) \right] } \\
\times e^{\left( \frac{\sigma^2_t}{\sigma^2} \right) \left( \nu^2_y + \nu^2_y \right) \left[ 1 - F_\epsilon (n - 2) \right] + \left( \frac{\sigma^2_t}{\sigma^2} \right) \nu^2 \left[ F_x (n - 2) - U_x F_\epsilon (n - 2) \right] } \\
\times e^{\left( \frac{\sigma^2_t}{\sigma^2} \right) \left[ F_\sigma (n - 2) - U_\sigma F_\epsilon (n - 2) \right] } \\
E_t \left[ V^{T}_{d1} (t + 1) \right] = e^{yt + \mu_y + \rho_y x_t + U_0 (n - 1) + U_x (n - 1) \rho_x x_t + U_\sigma (n - 1) \left[ \sigma^2 (1 - \rho_\sigma) + \rho_\sigma \sigma^2 \right] } \\
\times e^{\frac{1}{2} \left( U_x (n - 1) \nu^2_y \right) \left( \sigma^2_x + \frac{1}{2} (U_\sigma (n - 1) \nu^2_y) \right) + \frac{1}{2} \left( \nu^2_y + \nu^2_y \right) } \\
E_t \left[ V^{T}_{d2} (t + 1) \right] = e^{yt + \mu_y + \rho_y x_t + 2 F_0 (n - 1) } \\
\times e^{2 F_x (n - 1) \left[ \tau + \nu \right] + \rho_{x_t} x_t + \rho_{\sigma^2} (\sigma^2 - \sigma^2) } \\
\times e^{2 F_x (n - 1) \rho_x x_t + 2 F_\sigma (n - 1) \left[ \sigma^2 + \rho_\sigma \left( \sigma^2 - \sigma^2 \right) \right] } \\
\times e^{2 \sigma_t^2 \left( \nu^2_y + \nu^2_y \right) \left[ 1 - F_\epsilon (n - 1) \right] + 2 \sigma_t^2 \nu^2 \left[ F_x (n - 1) - U_x F_\epsilon (n - 1) \right] } \\
\times e^{2 \sigma_t^2 \left[ F_\sigma (n - 1) - U_\sigma F_\epsilon (n - 1) \right] } \\
E_t \left[ V^{T}_{d2} (t + 1) \right] = e^{yt + \mu_y + \rho_y x_t + 2 F_0 (n - 2) } \\
\times e^{2 F_x (n - 2) \left[ \tau + \nu \right] + \rho_{x_t} x_t + \rho_{\sigma^2} (\sigma^2 - \sigma^2) } \\
\times e^{2 F_x (n - 2) \rho_x x_t + 2 F_\sigma (n - 2) \left[ \sigma^2 + \rho_\sigma \left( \sigma^2 - \sigma^2 \right) \right] } \\
\times e^{2 \sigma_t^2 \left( \nu^2_y + \nu^2_y \right) \left[ 1 - F_\epsilon (n - 2) \right] + 2 \sigma_t^2 \nu^2 \left[ F_x (n - 2) - U_x F_\epsilon (n - 2) \right] } \\
\times e^{2 \sigma_t^2 \left[ F_\sigma (n - 2) - U_\sigma F_\epsilon (n - 2) \right] } \\
E_t \left[ V^{T}_{y} (t + 1) \right] = e^{yt + \mu_y + \rho_y x_t + 2 U_0 (n - 1) + 2 U_x (n - 1) \rho_x x_t } \\
\times e^{2 U_\sigma (n - 1) \left[ \sigma^2 (1 - \rho_\sigma) + \rho_\sigma \sigma^2 \right] } \\
\times e^{2 U^2_x (n - 1) \nu^2_x \sigma^2 + 2 U^2_x (n - 1) \nu^2 + 2 \sigma_t^2 \left( \nu^2_y + \nu^2_y \right) }
\[ E_t \left[ V_{d1}^T(t + 1) V_{d2}^T(t + 1) \right] = e^{2y_t + 2\mu_y + 2\rho_y x_t + F_0(n-1) + F_0(n-2)} \]
\[ \times e^{(F_x(n-1) + F_x(n-2))\left[\sigma_t + \rho_x x_t + \rho_{x\sigma}(\sigma_t^2 - \sigma^2)\right]} \]
\[ \times e^{(F_x(n-1) + F_x(n-2))\rho_x x_t + (F_{\sigma}(n-1) + F_{\sigma}(n-2))\left[\sigma_{t\sigma} + \rho_{\sigma}(\sigma_t^2 - \sigma^2)\right]} \]
\[ \times e^{\sigma_t^2(\nu_{yc} + \nu_y)^2\left[2 - F_{\ell}(n-1) - F_{\ell}(n-2)\right]^2} \]
\[ \times e^{\frac{\nu_y^2}{2} \left[ F_{x}(n-1) + F_{x}(n-2) - U_x \left( F_{\ell}(n-1) - F_{\ell}(n-2) \right) \right]^2} \]
\[ \times e^{\frac{\nu_{yc}^2}{2} \left[ F_{\sigma}(n-1) + F_{\sigma}(n-2) - U_{\sigma} \left( F_{\ell}(n-1) - F_{\ell}(n-2) \right) \right]^2} \]

\[ E_t \left[ V_{d2}^T(t + 1) V_{d2}^T(t + 1) \right] = e^{2y_t + 2\mu_y + 2\rho_y x_t + F_0(n-2) + U_0(n-1)} \]
\[ \times e^{F_x(n-1)\left[\sigma_t + \rho_x x_t + \rho_{x\sigma}(\sigma_t^2 - \sigma^2)\right]} \]
\[ \times e^{(F_x(n-2) + U_x(n-1))\rho_x x_t + (F_{\sigma}(n-2) + U_{\sigma}(n-1))\left[\sigma_{t\sigma} + \rho_{\sigma}(\sigma_t^2 - \sigma^2)\right]} \]
\[ \times e^{\sigma_t^2(\nu_{yc} + \nu_y)^2\left[2 - F_{\ell}(n-1)\right]^2} \]
\[ \times e^{\frac{\nu_y^2}{2} \left[ F_{x}(n-2) + U_x(n-1) - U_x F_{\ell}(n-1) \right]^2} \]

\[ E_t \left[ V_{d2}^T(t + 1) V_{d2}^T(t + 1) \right] = e^{2y_t + 2\mu_y + 2\rho_y x_t + F_0(n-2) + U_0(n-1)} \]
\[ \times e^{F_x(n-2)\left[\sigma_t + \rho_x x_t + \rho_{x\sigma}(\sigma_t^2 - \sigma^2)\right]} \]
\[ \times e^{(F_x(n-2) + U_x(n-1))\rho_x x_t + (F_{\sigma}(n-2) + U_{\sigma}(n-1))\left[\sigma_{t\sigma} + \rho_{\sigma}(\sigma_t^2 - \sigma^2)\right]} \]
\[ \times e^{\sigma_t^2(\nu_{yc} + \nu_y)^2\left[2 - F_{\ell}(n-2)\right]^2} \]
\[ \times e^{\frac{\nu_y^2}{2} \left[ F_{x}(n-2) + U_x(n-1) - U_x F_{\ell}(n-2) \right]^2} \]

\[ E_t \left[ V_{d2}^T(t + 1) V_{d2}^T(t + 1) \right] = e^{2y_t + 2\mu_y + 2\rho_y x_t + F_0(n-2) + U_0(n-1)} \]
\[ \times e^{F_x(n-2)\left[\sigma_t + \rho_x x_t + \rho_{x\sigma}(\sigma_t^2 - \sigma^2)\right]} \]
\[ \times e^{(F_x(n-2) + U_x(n-1))\rho_x x_t + (F_{\sigma}(n-2) + U_{\sigma}(n-1))\left[\sigma_{t\sigma} + \rho_{\sigma}(\sigma_t^2 - \sigma^2)\right]} \]
\[ \times e^{\sigma_t^2(\nu_{yc} + \nu_y)^2\left[2 - F_{\ell}(n-2)\right]^2} \]
\[ \times e^{\frac{\nu_y^2}{2} \left[ F_{x}(n-2) + U_x(n-1) - U_x F_{\ell}(n-2) \right]^2} \]

\[ E_t \left[ V_{d2}^T(t + 1) V_{d2}^T(t + 1) \right] = e^{2y_t + 2\mu_y + 2\rho_y x_t + F_0(n-2) + U_0(n-1)} \]
\[ \times e^{F_x(n-2)\left[\sigma_t + \rho_x x_t + \rho_{x\sigma}(\sigma_t^2 - \sigma^2)\right]} \]
\[ \times e^{(F_x(n-2) + U_x(n-1))\rho_x x_t + (F_{\sigma}(n-2) + U_{\sigma}(n-1))\left[\sigma_{t\sigma} + \rho_{\sigma}(\sigma_t^2 - \sigma^2)\right]} \]
\[ \times e^{\sigma_t^2(\nu_{yc} + \nu_y)^2\left[2 - F_{\ell}(n-2)\right]^2} \]
\[ \times e^{\frac{\nu_y^2}{2} \left[ F_{x}(n-2) + U_x(n-1) - U_x F_{\ell}(n-2) \right]^2} \]

**E.3. Equity Returns**

To derive the relation between returns on the EBIT claim and returns on the dividend claim, we solve for the exact solution in continuous time, and then use the solution as an approximation for our discrete time-setting. In particular, we notionally express the
returns to enterprise value, the stock, and the risk-free bond via
\[
\frac{dV_y + \delta_y V_y dt}{V_y} \equiv e^{r_y} - 1 = \mu_y dt + \sigma_y dz \\
\frac{dV_d + \delta_d V_d dt}{V_y} \equiv e^{r_d} - 1 = \mu_d dt + \sigma_d dz
\]
\[
\frac{dB}{B} = r dt. \tag{IA.72}
\]
Defining leverage via \( \frac{B}{V_y} \equiv e^\ell \), standard results generate the relations
\[
\mu_y - r = (1 - e^\ell) [\mu_s - r] \\
\sigma_y = (1 - e^\ell) \sigma_s. \tag{IA.73}
\]
From their definitions, we can express the log returns via
\[
r_y = \left( \mu_y - \frac{1}{2} \sigma_y^2 \right) dt + \sigma_y dz \\
r_d = \left( \mu_s - \frac{1}{2} \sigma_s^2 \right) dt + \sigma_s dz. \tag{IA.74}
\]
Combining the last four equations, we find that we can relate the log stock return \( r_d \) to the enterprise value return \( r_y \) via
\[
r_d - r_f = \left( \frac{1}{1 - e^\ell} \right) \left[ r_y - r_f + \frac{\sigma_y^2}{2} \left( 1 - \left( \frac{1}{1 - e^\ell} \right) \right) \right]. \tag{IA.75}
\]
Importantly, the stochastic components are related via
\[
r_d \big|_{stoch} = \left( \frac{1}{1 - e^\ell} \right) r_y \big|_{stoch}. \tag{IA.76}
\]

\section*{E.4. Bounding Leverage}

To guarantee that leverage remains within empirically observed limits, we assume that there are asset sales when leverage moves above the upper threshold, and that the funds raised are used to repurchase debt. All of this is accomplished at “fair market values,” so these asset sales have no impact on firm value.

For example, assume that a firm finds itself with leverage higher than some specified maximum value \( L_{max} \), which we set to 65\%. At this time, enterprise value and the value of debt satisfy
\[
V_y(y_t, x_t, \sigma_t) \approx e^{y_t + U_0 + U_x x_t + U_s \sigma_t^2} \\
B(\ell_t, y_t, x_t, \sigma_t) = e^{\ell_t + y_t + U_0 + U_x x_t + U_s \sigma_t^2}. \tag{IA.77}
\]
The firm acts to lower leverage from $L_t \equiv e^{\ell_t}$ to $L_{\text{max}}$. It does so by selling off a fraction of its EBIT-generating machine. In particular, the firm sells off the fraction that reduces EBIT from $Y_t \equiv e^{y_t}$ to $(Y_t - \Delta Y_t)$. Because the value of the EBIT claim is linear in $Y_t$, it follows that the value of this sale is

$$Sale_t = \Delta Y_t e^{U_0 + U_x x_t + U_\sigma \sigma_t^2}. \quad (IA.78)$$

Because the funds raised are used to reduce the debt, the new enterprise value and new debt value are

$$V_{y,\text{new}}(y_t, x_t, \sigma_t) \approx (Y_t - \Delta Y_t) e^{U_0 + U_x x_t + U_\sigma \sigma_t^2}$$
$$B_{\text{new}}(\ell_t, y_t, x_t, \sigma_t) \approx (Y_t e^\ell_t - \Delta Y_t) e^{U_0 + U_x x_t + U_\sigma \sigma_t^2}. \quad (IA.79)$$

Hence, we determine the size of $\Delta Y_t$ by choosing it so that the new leverage is equal to the maximum:

$$L_{\text{max}} = \frac{B_{\text{new}}(\ell_t, y_t, x_t, \sigma_t)}{V_{y,\text{new}}(y_t, x_t, \sigma_t)} = \left( \frac{Y_t e^\ell_t - \Delta Y_t}{Y_t - \Delta Y_t} \right). \quad (IA.80)$$

Solving for $\Delta Y_t$, we find

$$\Delta Y_t = Y_t \left( \frac{e^{\ell_t} - L_{\text{max}}}{1 - L_{\text{max}}} \right). \quad (IA.81)$$

In terms of the state variable $y_t$, we find

$$e^{y_{\text{new}}} = e^{y_t} - \Delta Y, \quad (IA.82)$$

implying that

$$y_{\text{new}} = \log (e^{y_t} - \Delta Y). \quad (IA.83)$$

An analogous argument holds for asset purchases at a lower leverage threshold $L_{\text{min}}$, which we set to 25%. In particular, if $L_t < L_{\text{min}}$, then the firm purchases EBIT-generating machinery at cost

$$Purchase_t = \Delta Y_t e^{U_0 + U_x x_t + U_\sigma \sigma_t^2}. \quad (IA.84)$$
The funds raised to make this purchase are obtained by issuing debt. Hence, the new enterprise value and debt value are

\[
V_{y,\text{new}}(y_t, x_t, \sigma_t) \approx (Y_t + \Delta Y_t) e^{U_a + U_x x_t + U_\sigma \sigma^2_t}
\]

\[
B_{\text{new}}(\ell_t, y_t, x_t, \sigma_t) \approx (Y_t e^{\ell_t} + \Delta Y_t) e^{U_a + U_x x_t + U_\sigma \sigma^2_t}.
\]  

(IA.85)

Hence, we determine the size of \(\Delta Y_t\) by choosing it so that the new leverage is equal to \(L_{\text{min}}\):

\[
L_{\text{min}} \equiv \frac{B_{\text{new}}(\ell_t, y_t, x_t, \sigma_t)}{V_{y,\text{new}}(y_t, x_t, \sigma_t)}
\]

\[
= \left( \frac{Y_t e^{\ell_t} + \Delta Y_t}{Y_t + \Delta Y_t} \right).
\]  

(IA.86)

Solving for \(\Delta Y_t\), we find

\[
\Delta Y_t = Y_t \left( \frac{L_{\text{min}} - e^{\ell_t}}{1 - L_{\text{min}}} \right).
\]  

(IA.87)

In terms of the state variable \(y_t\), we find

\[
e^{y_{\text{new}}} = e^{y_t} + \Delta Y,
\]  

(IA.88)

implying that

\[
y_{\text{new}} = \log (e^{y_t} + \Delta Y).
\]  

(IA.89)

F. Additional Results on the Long-Run Risk Model

F.1. Approximation Errors in the BKY Calibration

In the manuscript we closely follow the calibration of BKY because it matches several empirical asset pricing moments better than the original Bansal and Yaron (BY, 2004) calibration. However, we changed the persistence of the volatility process from \(\rho_\sigma = 0.999\) used in BKY to \(\rho_\sigma = 0.995\), because the accuracy of the approximation they use to solve the model deteriorates quickly when the persistence parameter \(\rho_\sigma\) is pushed above 0.995. Figure IA.1 reports the results from this analysis.
F.2. Comparative Statics: The Role of the Economic State

BBK show that the price of dividends, scaled by the current level of dividends, predicts dividend strip returns and this predictability is more pronounced for short-term strips than for the market as a whole. Here, we perform comparative statics with respect to the main driver of time-varying risk premia (i.e., stochastic volatility) and show that the model is qualitatively consistent with this result.

In the BY framework, the stock price is a decreasing function of volatility, and excess returns are an increasing function. To examine the impact of the economic state, in Figure IA.2 we plot the term structure of excess returns for dividend strips when the variance is one standard deviation above its mean. We see that short-horizon dividend strips increase their excess return by an additional 2.7%, whereas the excess returns on longer-horizon dividend strips also increase, but to a lesser extent. This result is mostly consistent with the findings of BBK that the price of dividends, scaled by the current level of dividends, predicts dividend strip returns, and that this predictability is more pronounced for short-term strips than for the market as a whole.

II. Habit Model

The pricing kernel, market price of risk, and EBIT dynamics follow

\[ m_{t+1} = -r_f - \frac{1}{2} \tilde{\theta}^2_{t+1} - \theta_{t+1} \tilde{\epsilon}_{m,t+1} \]  \hspace{1cm} (IA.90)
\[ \theta_{t+1} = \tilde{\theta} (1 - \rho_\theta) + \rho_\theta \theta_t - \sigma_\theta \tilde{\epsilon}_{m,t+1} \]  \hspace{1cm} (IA.91)
\[ \Delta y_{t+1} = \mu_y + \nu_{ym} \tilde{\epsilon}_{m,t+1} + \nu_y \tilde{\epsilon}_{y,t+1} \]  \hspace{1cm} (IA.92)

We approximate the log of enterprise value to EBIT via

\[ z_{y,t} \equiv \log \left( \frac{V_{y,t}}{Y_t} \right) \approx U_0 - U_\theta \theta_t. \]  \hspace{1cm} (IA.93)

We also approximate the log return to the EBIT claim via

\[ r_{y,t+1} \approx \kappa_{0y} + \kappa_{1y} z_{y,t+1} - z_{y,t} + \Delta y_{t+1}, \]  \hspace{1cm} (IA.94)
where \( y_t = \log Y_t \) and

\[
\kappa_{1y} = \left( \frac{e^{\bar{z_y}}}{1 + e^{\bar{z_y}}} \right) \quad (\text{IA.95})
\]

\[
\kappa_{0y} = \log(1 + e^{\bar{z_y}}) - \bar{z_y} \left( \frac{e^{\bar{z_y}}}{1 + e^{\bar{z_y}}} \right)
\]

\[= -\log(1 - \kappa_{1y}) - \kappa_{1y} \log \left( \frac{\kappa_{1y}}{1 - \kappa_{1y}} \right). \quad (\text{IA.96})
\]

We need to identify \( \{\bar{z_y}, \kappa_{0y}, \kappa_{1y}, U_0, U_\theta\} \). Again, we already have three equations: the definitions of \( \{\kappa_{0y}, \kappa_{1y}\} \) and the self-consistent condition

\[
\bar{z_y} = U_0 - U_\theta \beta. \quad (\text{IA.97})
\]

The last two conditions are determined from the following first-order condition:

\[
0 = E_t \left[ m_{t+1} + r_{y,t+1} \right] + \frac{1}{2} \text{Var}_t \left[ m_{t+1} + r_{y,t+1} \right]. \quad (\text{IA.98})
\]

Note that

\[
E_t \left[ m_{t+1} + r_{y,t+1} \right] = -r_f - \frac{1}{2} \rho_t^2 + \kappa_{0y} + \kappa_{1y} U_0 - \kappa_{1y} U_\theta \beta \left( 1 - \rho_\theta \right)
\]

\[-\kappa_{1y} U_\theta \rho_\theta \theta_t - U_0 + U_\theta \theta_t + \mu_y. \]

Furthermore, we have

\[
\left[ m_{t+1} + r_{y,t+1} \right] - E_t \left[ m_{t+1} + r_{y,t+1} \right] = \left( \nu y + \kappa_{1y} U_\theta \sigma_\theta - \theta_t \right) \bar{e}_{m,t+1} + \nu_y \bar{e}_{y,t+1}. \quad (\text{IA.99})
\]

Collecting terms linear and independent of \( \theta_t \), we find the two conditions

\[
0 = -\kappa_{1y} U_\theta \rho_\theta + U_\theta - \kappa_{1y} U_\theta \sigma_\theta - \nu_y m
\]

\[
0 = -r_f + \kappa_{0y} + \kappa_{1y} U_0 - \kappa_{1y} U_\theta \beta \left( 1 - \rho_\theta \right) - U_0 + \mu_y + \frac{1}{2} \nu_y^2 + \frac{1}{2} \kappa_{1y} U_\theta^2 \sigma_\theta^2
\]

\[+ \frac{1}{2} \nu_y^2 + \nu_y m \kappa_{1y} U_\theta \sigma_\theta. \]

Simplifying, this gives

\[
U_\theta = \left( \frac{\nu_y m - \kappa_{1y} \rho_\theta - \kappa_{1y} \sigma_\theta}{1 - \kappa_{1y} \rho_\theta - \kappa_{1y} \sigma_\theta} \right)
\]

\[
U_0 = \left( \frac{1}{1 - \kappa_{1y}} \right) \left[ -r_f + \kappa_{0y} - \kappa_{1y} U_\theta \beta \left( 1 - \rho_\theta \right) + \mu_y + \frac{1}{2} \nu_y^2 + \frac{1}{2} \kappa_{1y} U_\theta^2 \sigma_\theta^2
\]

\[+ \frac{1}{2} \nu_y^2 + \nu_y \kappa_{1y} U_\theta \sigma_\theta \right]. \]
A. Enterprise Value Return

The log-linear approximation implies that the excess return on enterprise value can be expressed as
\[
(r_{y,t+1} - r_f) = \kappa_{0y} + \kappa_{1y} z_{y,t+1} - z_{y,t} + \Delta y_{t+1} - r_f. \tag{IA.100}
\]
It is convenient to decompose the previous equation in terms of the predicted and unexpected components:
\[
E_t [r_{y,t+1} - r_f] = \kappa_{0y} + \kappa_{1y} U_0 - \kappa_{1y} U_\theta \left[ \theta (1 - \rho_{\theta}) + \rho_{\theta} \theta_t \right] - U_0 + U_\theta \theta_t + \mu_y - r_f
\]
\[
[ r_{y,t+1} - r_f ] - E_t [ r_{y,t+1} - r_f ] = (\kappa_{1y} U_\theta \sigma_{\theta} + \nu_{ym}) \hat{\epsilon}_{m,t+1} + \nu_y \hat{\epsilon}_{y,t+1}. \tag{IA.101}
\]

We define the log expected excess return on the EBIT claim as
\[
\overline{r}_{y,t} = \log E_t [ e^{(r_{y,t+1} - r_f)} ]
\]
\[
= \kappa_{0y} + \kappa_{1y} U_0 - \kappa_{1y} U_\theta \left[ \theta (1 - \rho_{\theta}) + \rho_{\theta} \theta_t \right] - U_0 + U_\theta \theta_t + \mu_y - r_f
\]
\[
+ \frac{1}{2} \left( \nu_{ym} + \kappa_{1y} U_\theta \sigma_{\theta} \right)^2 + \frac{1}{2} \nu_y^2. \tag{IA.102}
\]
This simplifies to
\[
\overline{r}_{y,t} = \nu_{ym} \theta_t \left( \frac{1 - \kappa_{1y} \rho_{\theta}}{1 - \kappa_{1y} \rho_{\theta} - \kappa_{1y} \sigma_{\theta}} \right). \tag{IA.103}
\]

We also define the volatility on the EBIT claim as
\[
\sigma_{y,t} = \sqrt{\log E_t \left[ e^{2(r_{y,t+1} - r_f)} \right] - 2 \log E_t \left[ e^{(r_{y,t+1} - r_f)} \right]}
\]
\[
= \sqrt{(\kappa_{1y} U_\theta \sigma_{\theta} + \nu_{ym})^2 + \nu_y^2}. \tag{IA.104}
\]

B. Term Structure of EBIT

Because log-EBIT follows an arithmetic Brownian motion process, the term structure of dividend expected growth rates is flat over all horizons \( n \):
\[
g_{y,n} = \frac{1}{n} \log \left( E_t \left[ e^{y_T - y_t} \right] \right)
\]
\[
= \mu_y + \frac{1}{2} \left( \nu_{ym}^2 + \nu_y^2 \right) \quad \forall n. \tag{IA.105}
\]
Similarly, the term structure of dividend volatilities is also flat over all horizons:

\[
\sigma_{y,n} = \sqrt{\frac{1}{n} \left[ \log E_t \left[ e^{2(y_T - y_t)} \right] - 2 \log E_t \left[ e^{y_T - y_t} \right] \right]}
\]

\[
= \sqrt{\left( \nu^2_{ym} + \nu^2_y \right)} \quad \forall n. \quad \text{(IA.106)}
\]

C. EBIT Strips

The date \( t \) claim to the EBIT strip \( e^{y_T} \) paid out at date \( T \) is defined as

\[
V_T^{T}(t, y_T, \theta_t) = E_t \left[ e^{(\sum_{i=1}^{n} m_{t+i}) + y_T} \right]. \quad \text{(IA.107)}
\]

Note that from the law of iterated expectations we have

\[
V_T^{T}(t-1, y_{t-1}, \theta_{t-1}) = E_{t-1} \left[ e^{(\sum_{i=0}^{n} m_{t+i}) + y_T} \right]
\]

\[
= E_{t-1} \left[ E_t \left[ e^{m_t + (\sum_{i=1}^{n} m_{t+i}) + y_T} \right] \right]
\]

\[
= E_{t-1} \left[ e^{m_t} V_y^{T}(t, y_t, \theta_t) \right]. \quad \text{(IA.108)}
\]

Again, since the state vector dynamics are affine, the solution is of the form

\[
V_T^{T}(t, y_t, \theta_t) = e^{y_T + U_0(n) - U_0(n)\theta_t} \quad \text{(IA.109)}
\]

\[
V_T^{T}(t-1, y_{t-1}, \theta_{t-1}) = e^{y_{T-1} + U_0(n+1) - U_0(n+1)\theta_{t-1}}. \quad \text{(IA.110)}
\]

The final conditions are \( U_0(0) = 0 \) and \( U_0(0) = 0 \).

Plugging equations (IA.109) and (IA.110) into equation (IA.108), performing the expectation, and then collecting terms independent of the state vector and linear in \( (\theta_t) \), we obtain the two recursive equations

\[
\theta_t : \quad U_0(n + 1) = U_0(n) (\rho_\theta + \sigma_\theta) + \nu_{ym} \quad \text{(IA.111)}
\]

\[
\text{indep} : \quad U_0(n + 1) = -r_f + \mu_y + U_0(n) - U_0(n)\bar{\theta}(1 - \rho_\theta)
\]

\[
+ \frac{1}{2} \left( \nu^2_y + \nu^2_{ym} \right) + \frac{\sigma_\theta^2}{2} U_0^2(n) + \nu_{ym} \sigma_\theta U_0(n). \quad \text{(IA.112)}
\]

To obtain the EBIT strip returns, define the date \( t \) one-period gross return on a strip that matures at date \( T \) via

\[
\tilde{R}_{y,t+1}^T \equiv e^{\tilde{y}_{T,y,t+1}}
\]

\[
= \frac{V_T^{T}(t+1, y_{t+1}, \theta_{t+1})}{V_T^{T}(t, y_t, \theta_t)}. \quad \text{(IA.113)}
\]
We can decompose \((\tilde{r}^{T}_{y,t+1} - r_f)\) into its predictable and unpredictable components via

\[
E_t \left[ \tilde{r}^{T}_{y,t+1} - r_f \right] = \mu_y + U_0(n - 1) - U_0(n - 1) \left[ \overline{\theta} (1 - \rho_x) + \rho_x \theta_t \right] - U_0(n) + U_0(n) \theta_t - r_f.
\]

\[
\tilde{r}^{T}_{y,t+1} - r_f - E_t \left[ \tilde{r}^{T}_{y,t+1} - r_f \right] = (\nu_{ym} + \sigma_x U_0(n - 1) \tilde{\epsilon}_{m,t+1} + \nu_y \tilde{\epsilon}_{y,t+1} - U_0(n) + U_0(n) \theta_t - r_f).
\]

The log expected excess returns for the EBIT strips are

\[
\bar{\tau}(V^T_y(t)) = \log E_t \left[ e^{(\tilde{r}^{T}_{y,t+1} - r_f)} \right] = \mu_y + U_0(n - 1) - U_0(n - 1) \left[ \overline{\theta} (1 - \rho_x) + \rho_x \theta_t \right] + U_0(n) \theta_t - r_f + \frac{1}{2} (\sigma_x U_0(n - 1) + \nu_{ym})^2 + \frac{1}{2} \nu_y^2. \tag{IA.114}
\]

This simplifies to

\[
\bar{\tau}(V^T_y(t)) = \theta_t \left( \sigma_x U_0(n - 1) + \nu_{ym} \right). \tag{IA.115}
\]

We also define dividend strip volatility as

\[
\sigma(V^T_y(t)) = \sqrt{\log E_t \left[ e^{2(\tilde{r}^{T}_{y,t+1} - r_f)} \right] - 2 \log E_t \left[ e^{(\tilde{r}^{T}_{y,t+1} - r_f)} \right]} = \sqrt{\left( \sigma_x U_0(n - 1) + \nu_{ym} \right)^2 + \nu_y^2}. \tag{IA.116}
\]

D. Leverage Dynamics

Recall from equation (IA.93) that

\[
V_y(y_t, \theta_t) \approx e^{y_t + U_0 - U_0 \theta_t}. \tag{IA.117}
\]

Here, we assume that at all dates \(t\) the firm issues riskless debt that matures at date \((t + dt)\) with present value equal to

\[
B(\ell_t, y_t, \theta_t) = e^{\ell_t + y_t + U_0 - U_0 \theta_t} \approx e^{\ell_t} V_y(y_t, \theta_t). \tag{IA.118}
\]

We interpret \(e^{\ell_t} \approx \frac{B(\ell_t, y_t, \theta_t)}{V_y(y_t, \theta_t)}\) as the leverage of the firm. Since it is riskless, the firm must pay \(e^{r_f} B(\ell_t, y_t, \theta_t)\) at date \((t + 1)\). It does so by issuing at this time debt with face value
\[ B(\ell_{t+1}, y_{t+1}, \theta_{t+1}), \text{with all residual cash flows paid out as dividends. As such, dividends} \]
\[ D(t + 1) \text{ paid out at date } (t + 1) \text{ are} \]
\[ D(t + 1) = B(\ell_{t+1}, y_{t+1}, \theta_{t+1}) - e^{r_f} B(\ell_t, y_t, \theta_t) + e^{y_{t+1}} \]
\[ \equiv D_1(t + 1) - D_2(t + 1) + e^{y_{t+1}}. \]  
\[ \text{(IA.119)} \]

We specify log-leverage dynamics via
\[ \ell_{t+1} = \bar{\ell} + \rho_t (\ell_t - \bar{\ell}) + \rho_{\theta} (\theta_t - \bar{\theta}) - \nu_{y_m} \tilde{\epsilon}_{m,t+1} - \nu_y \tilde{\epsilon}_{y,t+1} - U_\sigma \tilde{\epsilon}_{\sigma,t+1}. \]  
\[ \text{(IA.120)} \]

E. Dividend Strips

Here we provide a closed-form expression for the price of dividend strips, defined as
\[ V^T_d(t, \ell_t, y_t, \theta_t) = E_t \left[ e^{\sum_{i=1}^n m_{t+i}} D(T) \right] \]
\[ = E_t \left[ e^{\sum_{i=1}^n m_{t+i}} (D_1(T) - D_2(T) + e^{y_T}) \right]. \]  
\[ \text{(IA.121)} \]

The price of dividend strips is the sum of three terms, each of which can be expressed in an exponential-affine form. The first term is
\[ V^T_{d,1}(t, \ell_t, y_t, \theta_t) = E_t \left[ e^{\sum_{i=1}^n m_{t+i}} B(\ell_T, y_T, \theta_T) \right]. \]  
\[ \text{(IA.122)} \]

The second term is
\[ V^T_{d,2}(t, \ell_t, y_t, \theta_t) = E_t \left[ e^{\sum_{i=1}^{n-1} m_{t+i}} B(\ell_{T-1}, y_{T-1}, \theta_{T-1}) e^{r_f} \right] \]
\[ = E_t \left[ e^{\sum_{i=1}^{n-1} m_{t+i}} B(\ell_{T-1}, y_{T-1}, \theta_{T-1}) E_{T-1} \left[ e^{m_T + r_f} \right] \right] \]
\[ = E_t \left[ e^{\sum_{i=1}^{n-1} m_{t+i}} B(\ell_{T-1}, y_{T-1}, \theta_{T-1}) \right] \]
\[ = V^T_{d,1}(t, \ell_{t-1}, y_{t-1}, \theta_{t-1}). \]  
\[ \text{(IA.123)} \]

The third term is the claim to the EBIT strip, \( V^T_y(t, y_t, \theta_t) \).

From the law of iterated expectations, the first term satisfies
\[ V^T_{d,1}(t - 1, \ell_{t-1}, y_{t-1}, \theta_{t-1}) = E_{t-1} \left[ e^{\sum_{i=0}^{n-1} m_{t+i}} B(\ell_T, y_T, \theta_T) \right] \]
\[ = E_{t-1} \left[ E_t \left[ e^{m_t + \sum_{i=1}^{n-1} m_{t+i}} B(\ell_T, y_T, \theta_T) \right] \right] \]
\[ = E_{t-1} \left[ e^{m_t} V^T_{d,1}(t, \ell_t, y_t, \theta_t) \right]. \]  
\[ \text{(IA.124)} \]
Because the state vector dynamics are affine, we know the solution is of the form

\[
V_{d,1}^T(t, \ell, t, y, \theta) = e^{\gamma_t + F_0(n) + F_t(n)\ell - F_0(\theta)}
\]  

\[
V_{d,1}^T(t-1, \ell, t, y, \theta) = e^{\gamma_{t-1} + F_0(n+1) + F_t(n+1)\ell - F_0(\theta)}
\]

The final conditions are \(F_0(0) = U_y, F_t(0) = 1,\) and \(F_0(0) = U_y.\)

Plugging equations (IA.125) and (IA.126) into equation (IA.124), performing the expectation, and then collecting terms independent of the state vector and linear in \((\ell, \theta),\) we obtain the three recursive equations

\[
\begin{align*}
\ell_t & : F_t(n+1) = \rho_t F_t(n) \tag{IA.127} \\
\theta_t & : F_\theta(n+1) = -\rho_{\theta\theta} F_t(n) + \rho_\theta F_\theta(n) + \nu y_m \\
\text{indep} & : F_0(n+1) = -r_f + \mu_y + F_0(n) + F_t(n)\bar{1}(1 - \rho_t) - \rho_{\theta y} F_\theta(n) \\
& \quad - (1 - \rho_\theta)\bar{\sigma} F_\sigma(n) + \frac{\nu^2}{2} [1 - F_\theta(n)]^2 \\
& \quad + \frac{1}{2} [\nu y_m - F_t(n)\nu y_m - F_\theta(n)U_y\sigma_y + F_\theta(n)\sigma_y]^2.
\end{align*}
\]

To obtain the return on dividend strips, we define the gross excess return on dividend strips as

\[
\frac{\bar{R}_{d,1}^T}{R_f} = \frac{V_{d,1}^T}{V_{d,1}^T}
\]

\[
= \left( \frac{1}{V_{d,1}^T R_f} \right) \left[ V_{d,1}^T(t+1) - V_{d,2}^T(t+1) + V_y^T(t+1) \right]. \tag{IA.128}
\]

The expectation has three terms:

\[
E_t \left[ V_{d,1}^T(t+1) \right] = e^{\gamma_t + \mu_y + F_0(n-1) + F_t(n-1)\bar{1} + \rho_t (\ell_t - \bar{1}) + \rho_{\theta\theta}(\theta_t - \bar{\theta})} - F_\theta(n-1)\bar{1} - F_\theta(n-1)\sigma_y^2 + \frac{\nu^2}{2} [\nu y_m - F_t(n-1)\nu y_m - F_\theta(n-1)\sigma_y]^2
\]

\[
E_t \left[ V_{d,2}^T(t+1) \right] = e^{\gamma_t + \mu_y + F_0(n-2) + F_t(n-2)\bar{1} + \rho_t (\ell_t - \bar{1}) + \rho_{\theta\theta}(\theta_t - \bar{\theta})} - F_\theta(n-2)\bar{1} - F_\theta(n-2)\sigma_y^2 + \frac{\nu^2}{2} [\nu y_m - F_t(n-2)\nu y_m - F_\theta(n-2)\sigma_y]^2
\]

\[
E_t \left[ V_y^T(t+1) \right] = e^{\gamma_t + \mu_y + U_0(n-1) - U_\theta(y-1)\bar{1} - \rho_\theta \theta_t + \frac{\nu^2}{2} (U_\theta(n-1)\sigma_y + \nu y_m)^2 + \frac{\nu^2}{2}.
\]
The second moments are

\[
E_t \left[ (V_{d_1}^T(t+1))^2 \right] = e^{2y_t + 2\mu_y + F_0(n-1) + 2F_t(n-1)\left[\bar{\tau} + \rho_t (\ell_t - \bar{\tau}) + \rho_{t\phi} (\theta_t - \bar{\phi})\right]}
\times e^{-2F_0(n-1)\left[\bar{\tau}(1 - \rho_y) + \rho_y \theta_t\right]} + 2\nu_y^2 \left[1 - F_t(n-1)\right]^2
\times e^{2 \left[\nu_{ym} - \nu_{ym} F_t(n-1) - U_y \sigma_y F_t(n-1) + F_0(n-1)\sigma_y\right]^2}
\]

\[
E_t \left[ (V_{d_2}^T(t+1))^2 \right] = e^{2y_t + 2\mu_y + 2F_0(n-2) + 2F_t(n-2)\left[\bar{\tau} + \rho_t (\ell_t - \bar{\tau}) + \rho_{t\phi} (\theta_t - \bar{\phi})\right]}
\times e^{-2F_0(n-2)\left[\bar{\tau}(1 - \rho_y) + \rho_y \theta_t\right]} + 2\nu_y^2 \left[1 - F_t(n-2)\right]^2
\times e^{2 \left[\nu_{ym} - \nu_{ym} F_t(n-2) - U_y \sigma_y F_t(n-2) + F_0(n-2)\sigma_y\right]^2}
\]

\[
E_t \left[ (V_{y}^T(t+1))^2 \right] = e^{2y_t + 2\mu_y + 2U_0(n-1) - 2U_y(n-1)\left[\bar{\tau}(1 - \rho_y) + \rho_y \theta_t\right] + 2(U_t(n-1)\sigma_y + \nu_{ym})^2 + 2\nu_y^2}
\]

\[
E_t \left[ V_{d_1}^T(t+1) V_{d_2}^T(t+1) \right] = e^{2y_t + 2\mu_y + F_0(n-1) + F_0(n-2) + (F_t(n-1) + F_t(n-2))\left[\bar{\tau} + \rho_t (\ell_t - \bar{\tau}) + \rho_{t\phi} (\theta_t - \bar{\phi})\right]}
\times e^{-\left(F_0(n-1) + F_0(n-2)\right)\left[\bar{\tau}(1 - \rho_y) + \rho_y \theta_t\right] + \left(\nu_y^2\right) \left[2 - F_t(n-1) - F_t(n-2)\right]^2}
\times e^{2 \left[\nu_{ym} - \nu_{ym} + \nu_{ym} \sigma_y(F_t(n-1) + F_t(n-2)) + \nu_{ym} \sigma_y(F_t(n-1) + F_t(n-2))\right]^2}
\]

\[
E_t \left[ V_{d_1}^T(t+1) V_{y}^T(t+1) \right] = e^{2y_t + 2\mu_y + F_0(n-1) + U_0(n-1) + F_t(n-1)\left[\bar{\tau} + \rho_t (\ell_t - \bar{\tau}) + \rho_{t\phi} (\theta_t - \bar{\phi})\right]}
\times e^{-\left(F_0(n-1) + U_0(n-1)\right)\left[\bar{\tau}(1 - \rho_y) + \rho_y \theta_t\right] + \left(\nu_y^2\right) \left[2 - F_t(n-1)\right]^2}
\times e^{2 \left[\nu_{ym} - \nu_{ym} + \nu_{ym} \sigma_y(F_t(n-1)) + \nu_{ym} \sigma_y(F_t(n-1))\right]^2}
\]

\[
E_t \left[ V_{d_2}^T(t+1) V_{y}^T(t+1) \right] = e^{2y_t + 2\mu_y + F_0(n-2) + U_0(n-1) + F_t(n-2)\left[\bar{\tau} + \rho_t (\ell_t - \bar{\tau}) + \rho_{t\phi} (\theta_t - \bar{\phi})\right]}
\times e^{-\left(F_0(n-2) + U_0(n-1)\right)\left[\bar{\tau}(1 - \rho_y) + \rho_y \theta_t\right] + \left(\nu_y^2\right) \left[2 - F_t(n-2)\right]^2}
\times e^{2 \left[\nu_{ym} - \nu_{ym} + \nu_{ym} \sigma_y(F_t(n-2)) + \nu_{ym} \sigma_y(F_t(n-2))\right]^2}.
\]
III. Additional Empirical Results

In the main text, we investigate the properties (variance ratios) of aggregate dividends of publicly and privately traded firms (that is, using Flow of Funds data). To help establish the robustness of the analysis, here we investigate the properties of three alternative measures of aggregate dividends using data from publicly traded firms only. The analysis here confirms that the downward slope of the term structure of aggregate dividend volatility is a robust feature of the data, and we explicitly link this feature of the data to the negative serial correlation of the aggregate dividend series. In addition, we report the following additional empirical analysis and robustness checks to support the economic mechanisms highlighted in the main text: (i) We document a strong positive link between the aggregate leverage ratio and the ratio of bondholders’ and stockholders’ payout (which captures the co-integration residual between the two payout variables); (ii) We show that leverage predicts future excess returns with a positive slope (the risk premium on equity is high when leverage is high); and (iii) We show that the net (which excludes short-term assets such as cash), not just gross as reported in the main text, aggregate leverage ratio of public and privately traded firms is also stationary.

A. Data

As in the main text, we perform the analysis using annual data to avoid the seasonality in dividend payments. The use of an annual dividend series implies that we need to take a stance on how dividends received within a particular year are reinvested. We consider two alternative reinvestment strategies. In the first strategy, we assume the monthly dividends are reinvested in the aggregate stock market. As in van Binsbergen and Koijen (2010), we refer to this dividend series as market-invested dividends. This measure of dividends is by far the most common in the dividend growth and return forecasting literature.¹ In the second strategy, we invest the monthly dividends in cash and obtain a time series of annual dividends that we call cash-invested dividends. As shown by van Binsbergen and Koijen (2010) and Chen (2009), the two dividend series have different time-series properties in the post-war sample period.

We obtain the data for the two dividend series from Long Chen’s webpage (the data are used in Chen (2009), and we extend the data to 2010 by replicating Chen’s procedure). We use this data set because it covers a long sample period from 1873 to 2008, thus covering the pre-Center for Research in Security Prices (CRSP) period.
Focusing on this long sample allows us to obtain more robust results. To construct the two dividend series, Chen (2009) combines the pre-CRSP data compiled by Schwert (1990) with the data from the CRSP (NYSE/Amex/NASDAQ) value-weighted market portfolio at the monthly frequency. We refer the reader to Chen (2009) for additional details on the construction of the two dividend series.

In addition to the previous two dividend series, we investigate a third alternative measure of dividends that includes share repurchases. The data for this alternative dividend series are available from Motohiro Yogo’s webpage (the data are used in Gomes, Kogan, and Yogo (2009), and cover a relatively shorter sample period from 1927 to 2007). Examining this alternative definition of dividends is motivated by the observation that firms have increased the fraction of payouts to shareholders via repurchase programs compared to dividends in recent years (Fama and French (2001), Grullon and Michaely (2002)). Still, as discussed in Lettau and Ludvigson (2005), large firms with high earnings have continued to increase traditional dividend payouts over time (DeAngelo, DeAngelo, and Skinner (2004)). The impact on aggregate dividends is therefore unclear. To show that our main findings are not altered by adjusting dividends to account for share repurchase activity since 1971, we consider a dividend series augmented with equity repurchases using Compustat’s statement of cash flows. All nominal quantities are deflated by the consumer price index (CPI), which is available from Robert Shiller’s webpage.

B. Dividend Variance Ratios of Publicly Traded Firms

The first two panels in Table IA.I reports the VR test results for the three definitions of aggregate dividends of publicly traded firms. Consistent with the results reported in the main text, the table shows that dividends do not follow a random walk. The VR test statistic decreases strongly with horizon for the three alternative dividend measures, that is, the variance of dividend growth is significantly smaller at long horizons than at short horizons.

At a fundamental level, the finding that the dividend variance decreases with horizon reflects negative serial correlation in the dividend growth series. To show this formally, we consider a simple econometric approach based on a linear regression. Specifically, we investigate if past values of dividend growth help predict future dividend growth by running a regression of the form \[ \Delta d_t = a + \sum_{i=1}^{5} b_i \Delta d_{t-i}. \] The results reported in Table IA.II show that past values of dividend growth help predict future dividends. The slope
coefficients on the lagged values of dividend growth are negative. It is this pattern of negative autocorrelation that drives the decreasing pattern of dividend volatility across horizon.

C. Leverage, Dividends, and Bondholders Payout

The main economic mechanism in the model ties the aggregate firm leverage ratio to the aggregate bondholder and stockholder payouts. The stationarity of the leverage ratio implies stationarity of the bondholder and stockholder payouts (i.e., these two payouts should be co-integrated). Thus, the aggregate leverage ratio should be related to the co-integration residual of the bondholders’ and stockholders’ payouts. Here, we provide empirical evidence for this link in the data.

Figure IA.3 plots the time series of the aggregate leverage ratio and the aggregate bondholder-shareholder payout ratio (this ratio is naturally closely related to the co-integration between the two payouts). Both variables are standardized (mean zero and unit standard deviation) to facilitate the analysis. The two series are strongly positively contemporaneously correlated, with a correlation of +50%. Importantly, the two series also share a strong common cycle (low frequency correlation). Thus, this figure shows that when the ratio of bondholder to stockholder payout is below the mean (here, below zero), the leverage ratio tends to be below its mean as well (here, below zero). Given mean-reversion in the leverage ratio, we expect to see leverage increase in the future if it is currently below the mean. As leverage increases, the bondholder-shareholder payout ratio increases as well, consistent with Figure IA.3.

D. Leverage and Time-Varying Risk Premia

According to the discussion in the main text, time-varying leverage implies time-variation in the aggregate risk premium (return on equity). In this section, we provide empirical support for this additional prediction of the model(s). We confirm empirically the prediction that leverage forecasts future excess returns with a positive slope.

To examine the relationship between leverage and risk premiums, we run standard short- and long-horizon predictive regressions (e.g., Fama and French (1989), and Lettau and Ludvigson (2002)). The dependent variable in the predictive regression is the $T$-year cumulated log aggregate stock market return, in which $T$ is the forecast horizon ranging from one year to 20 years. Specifically, we run a long-horizon forecasting regression of
the form

\[ \sum_{h=1}^{T} y_{t+h} = a + b \text{Lev}_t + \varepsilon_{it}, \]  

(IA.129)

where \( y_t = r_{st} - r_{ft} \), \( r_{st} \) is the log aggregate stock market return, \( r_{ft} \) is the log risk-free rate, and \( \text{Lev}_t \) is the current value of the log aggregate leverage ratio. For each horizon \( T = 1, \ldots, 20 \), we report the estimated slope associated with leverage, the corresponding \( t \)-statistic, and the regression \( R^2 \). In computing the \( t \)-statistic of the slope coefficient, we use standard errors corrected for autocorrelation per Newey and West (1987) with lag equal to three years plus the overlapping period, and a GMM correction for heteroskedasticity.

Table IA.III reports the results of the long-horizon predictability regressions. The results show that leverage forecasts future excess returns (risk premia) with a positive slope. Thus, periods with high leverage are periods with high risk premia. The regression \( R^2 \) increases from 4.6% at the one-year horizon to 29.3% at the 20-year horizon.

E. Stationarity of Gross and Net Leverage

In the main text we show that the aggregate gross leverage ratio is stationary. Here, we show that aggregate net leverage (which excludes short-term assets such as cash) exhibits similar stationary behavior. See also Wright (2004) for a similar analysis (our results are an update of Wright’s analysis).

Figure IA.4 plots the time series of the aggregate gross and net leverage ratio of public and private traded firms (Flow of Funds data). Although the level of the two series is different (net leverage is naturally smaller), both series appear to be equally stationary, consistent with the main mechanism in the model.
REFERENCES


Notes

1A noncomprehensive list of studies that use this measure of dividends includes Lettau and Ludvigson (2005), Cochrane (2008), and Lettau and Van Nieuwerburgh (2008).
Table IA.I
Dividend Variance Ratios

In the main article, the dividend variance ratio test demonstrates that dividend volatility drops significantly with horizon in the data. This table reports the variance ratios of three alternative definitions of dividends of publicly traded firms. $\sigma^T_{D,i}$ are the per-year standard deviation of the growth rate of variable dividends, for dividend volatility definition $i = 1, 2$. VR is the standard variance ratio using the formula in Lo and MacKinlay (1988). The data for dividend definitions 1 and 2 are annual from 1873 to 2010, and the data for dividend definition 3 are annual from 1927 to 2007.

<table>
<thead>
<tr>
<th>Horizon (years)</th>
<th>Diff</th>
<th>Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

Dividend definition 1: Market-invested dividends

| $\sigma^T_{D,1}$ | 15.67 | 15.12 | 12.94 | 11.16 | 9.39 | 8.16 | 7.87 | 7.15 | 7.51 | 8.52 |
| $\sigma^T_{D,2}$ | 15.95 | 15.31 | 12.15 | 10.47 | 9.01 | 7.66 | 7.24 | 6.91 | 8.28 | 9.04 |
| VR | 1.00 | 0.93 | 0.70 | 0.52 | 0.37 | 0.29 | 0.28 | 0.28 | – | – |

Dividend definition 2: Cash-invested dividends

| $\sigma^T_{D,1}$ | 13.29 | 13.89 | 12.34 | 10.53 | 8.98 | 7.62 | 7.62 | 6.69 | 5.68 | 6.60 |
| $\sigma^T_{D,2}$ | 13.41 | 13.80 | 11.82 | 10.17 | 8.71 | 7.22 | 7.17 | 6.8 | 6.20 | 6.62 |
| VR | 1.00 | 1.09 | 0.88 | 0.63 | 0.48 | 0.36 | 0.38 | 0.32 | – | – |

Dividend definition 3: With equity repurchases

| $\sigma^T_{D,1}$ | 13.65 | 14.15 | 13.64 | 10.65 | 7.61 | 7.27 | 6.87 | 5.41 | 6.39 | 8.24 |
| $\sigma^T_{D,2}$ | 13.45 | 14.26 | 14.04 | 10.60 | 7.25 | 7.11 | 7.16 | 5.20 | 6.35 | 8.25 |
| VR | 1.00 | 1.09 | 1.04 | 0.55 | 0.34 | 0.32 | 0.32 | 0.21 | – | – |
Table IA.II
Dividend Autocorrelation / Predictability

This table presents predictability regressions of real dividend growth on lagged values of dividend growth ($\Delta d_t = a + \sum_{i=1}^{5} b_i \Delta d_{t-i}$). The data for dividend definitions 1 and 2 are annual from 1930 to 2010, and the data for dividend definitions 3 and 4 are annual from 1873 to 2008.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b₁</th>
<th>b₂</th>
<th>b₃</th>
<th>b₄</th>
<th>b₅</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow of Funds: Public and Private Firms</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Dividend 1: Cash dividends</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>7.74</td>
<td>-0.43</td>
<td>-0.36</td>
<td>-0.33</td>
<td>-0.16</td>
<td>-0.08</td>
<td>13.27</td>
</tr>
<tr>
<td>[t]</td>
<td>2.84</td>
<td>-2.22</td>
<td>-2.26</td>
<td>-2.82</td>
<td>-0.99</td>
<td>-0.53</td>
<td></td>
</tr>
<tr>
<td><strong>Dividend 2: Cash dividends + equity repurchases</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>6.58</td>
<td>-0.09</td>
<td>-0.24</td>
<td>-0.15</td>
<td>-0.11</td>
<td>-0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>[t]</td>
<td>2.01</td>
<td>-0.79</td>
<td>-2.28</td>
<td>-1.16</td>
<td>-0.74</td>
<td>-0.54</td>
<td></td>
</tr>
<tr>
<td><strong>CRSP+Schwert: Public Firms</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Dividend 3: Market-invested dividends</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>5.99</td>
<td>-0.27</td>
<td>-0.26</td>
<td>-0.14</td>
<td>-0.18</td>
<td>-0.25</td>
<td>8.63</td>
</tr>
<tr>
<td>[t]</td>
<td>2.78</td>
<td>-2.54</td>
<td>-2.75</td>
<td>-1.12</td>
<td>-1.75</td>
<td>-1.79</td>
<td></td>
</tr>
<tr>
<td><strong>Dividend 4: Cash-invested dividends</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>3.69</td>
<td>0.06</td>
<td>-0.30</td>
<td>-0.06</td>
<td>-0.17</td>
<td>-0.07</td>
<td>5.16</td>
</tr>
<tr>
<td>[t]</td>
<td>2.31</td>
<td>0.64</td>
<td>-2.11</td>
<td>-0.40</td>
<td>-0.75</td>
<td>-0.59</td>
<td></td>
</tr>
</tbody>
</table>
Table IA.III
Leverage and Time-Varying Risk Premia

This table reports results from the following long-horizon predictability regression:

$$\sum_{h=1}^{T} y_{t+h} = a + b\text{Lev}_t + \varepsilon_{it},$$

where $y_t = r_{st} - r_{ft}$, $r_{st}$ is the log aggregate stock market return, $r_{ft}$ is the log risk-free rate, and $\text{Lev}_t$ is the current value of the log aggregate leverage ratio. $T$ is the forecast horizon in years. The table reports the OLS estimate of the slope coefficient associated with $\text{Lev}_t$, Slope, the Newey-West (1987) corrected $t$-statistic, $[t]$, and the adjusted $R^2$. The sample is annual from 1930 to 2010.

<table>
<thead>
<tr>
<th>Regressor</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Lev}_t$</td>
<td>Slope</td>
<td>0.20</td>
<td>0.38</td>
<td>0.54</td>
<td>0.61</td>
<td>0.79</td>
<td>1.03</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td>$[t]$</td>
<td>2.67</td>
<td>2.59</td>
<td>2.27</td>
<td>2.18</td>
<td>2.50</td>
<td>2.76</td>
<td>2.73</td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
<td>4.62</td>
<td>10.72</td>
<td>16.05</td>
<td>15.71</td>
<td>19.50</td>
<td>24.73</td>
<td>26.69</td>
</tr>
</tbody>
</table>
Figure IA.1. Approximation errors. The figures report the fractional error between two estimates for the stock price for the cases $\rho = 0.995$ and $\rho = 0.999$: one estimate directly estimates the stock price $V_{d,t}$ via equation (IA.25) and the other estimates the stock price via the sum of dividend strips $\sum V_{d,t}^T$ via equation (IA.53). The first figure sets $\sigma = \bar{\sigma}$ and then reports the fractional error as a function of $x$. The second figure sets $x = 0$ and then reports the fractional error as a function of $\sigma_t$. 
Figure IA.2. Impact of economic state. The left panel shows the term structure of dividend volatilities, and the right panel shows the term structure of excess returns on dividend strips, in the baseline calibration of the modified BY framework evaluated at the long-run mean of volatility, and in the baseline calibration with the current volatility evaluated at one standard deviation above its mean.
Figure IA.3. Leverage ratio and bondholders’ and stockholders’ payouts. This figure provides a time-series plot of the aggregate leverage ratio and the ratio of bondholders’ payout to stockholders’ payout. Both variables are standardized (mean zero and unit standard deviation). The sample is annual from 1930 to 2010.
Figure IA.4. **Gross versus net leverage ratio.** This figure provides a time-series plot of the aggregate gross and net leverage (exclude short-term assets) ratio. The sample is annual from 1930 to 2010.