Joint Rate and Power Control for Coded Cognitive Radio Networks

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Abstract—A cross-layer design in the presence of an underlying indeterminacy of propagation channels and interference levels is formulated for wireless cognitive networks, where the nodes are allowed to perform network coding. In lieu of full inter-system cooperation, statistics of signal-to-noise-plus-interference ratios (SINRs) and interference levels are leveraged to obtain optimal end-to-end session rates, network code design variables, and transmit-powers, while sharing spectral resources with incumbent primary systems in an underlay setup. Solution of the formulated optimization problem is hard to approach, mostly because of a lack of tractability of the resultant SINR distributions. Nonetheless, suitable approximations are employed to obtain an approximate convex problem, amenable to efficient solution. Numerical tests verify the ability of the proposed scheme to adapt network and physical layer parameters to the propagation environment.

Index Terms—Cognitive radio networks, network coding, power control, outage probability, cross-layer optimization.

I. INTRODUCTION

Judicious utilization of information provided by spatio-temporal sensing is critical for permeating the benefits of cognition at medium access control (MAC) and physical (PHY) layers through the entire protocol stack of wireless cognitive radio (CR) networks. Awareness of primary user (PU) activity and propagation medium characteristics not only provides tangible enhancements in link-reliability control, but also enables quality-of-service-assured operation in the presence of dynamics and uncertainties in the CR deployment. To this end, cross-layer approaches are well-suited for designing resource allocation schemes that rely on the sensing result [1], and facilitate the inclusion of capacity achieving protocols such as network coding [2], [3].

Nevertheless, pertinent approaches in CR networking can not rely on the accumulated knowledge in conventional ad hoc networks, but rather have to account for the peculiarities of hierarchical access schemes [4] and autonomous interference management. Specifically, because of the inherent lack of explicit inter-system cooperation, both signal-to-noise-plus-interference ratios (SINRs) and interference levels inflicted to PU receivers are uncertain. This, in turn, renders optimization of end-to-end throughput strenuous, as the underlying PHY layer rates are random.

In this context, channel uncertainty due to small-scale fading was considered in [5], where the sum of physical-layer data rates was maximized under outage probability constraints on the CR-to-CR channels. Besides small-scale fading, randomness arising from log-normal shadowing was also included in the physical-layer utility maximization framework proposed in [6]. The present paper considers the cross-layer design problem of jointly optimizing power and rate allocations in CR networks in the presence of channel uncertainty induced by both shadowing and small-scale fading. Furthermore, CR nodes are allowed to perform network coding, which is already a popular protocol for data dissemination in resource-constrained wireless multi-hop networks. Optimal integration of network coding in cross-layer optimization approaches with perfect channel state information has been studied in [3]. The emphasis here is on incorporating channel uncertainty-aware CR-specific constraints directly into the optimization formulation, so as to yield a resource allocation algorithm rooted at the PHY layer. Towards this end, the focus will be placed on delay-limited CR traffic, where quality of service (QoS) requirements are severe, and channel outages are common.

The primary challenge encountered when designing systems with outage probability constraints is that the resulting problem formulations are too complex for realistic channel distributions [7]. Section II formulates a joint coding/routing rate and power allocation problem, which maximizes the network-wide utility while constraining the outage probabilities and average interference to the PU networks. Since the formulated problem is non-convex and difficult to handle, Section III proposes a set of approximations that result in a convex problem with realistic system parameters such as fading variance. The problem formulation is also extended to the case when only broadcast transmissions are used with network coding in Section IV. Finally Section V provides numerical tests depicting rate allocations in realistic environments.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Physical and medium access control layer model

Consider a wireless CR network modeled as a directed graph \((\mathcal{N}, \mathcal{E})\), where \(\mathcal{N}\) collects the CR nodes, located at \(\{x_n\}_{n=1}^{N}\), and \(\mathcal{E}\) the set of CR-to-CR links. Leveraging the spectrum awareness provided by sensing schemes [8], [9], CRs employ power control to share a set of sub-bands \(\mathcal{F}\) with an incumbent PU system in an underlay setup [4].

Let \(g_{x_i \rightarrow x_j}\) denote the channel gain of link \((i, j)\) on (sub-
(1) band \( f \in \mathcal{F} \), which can be expressed as [10]
\[
g_{\xi,k} \rightarrow x_j = g_i ||\xi - x_j||^\eta + s^f_{\xi,k} \rightarrow x_j | h^f_{\xi,k} \rightarrow x_j|^2
\]
where \( g_i \) collects antenna, coding, and other propagation gains, \( \eta \) is the path loss exponent, \( s^f_{\xi,k} \rightarrow x_j \) the shadow fading, and \( h^f_{\xi,k} \rightarrow x_j \) the small-scale fading. Shadowing varies according to a log-normal distribution with \( 10 \log_{10}(s^f_{\xi,k} \rightarrow x_j) \sim \mathcal{N}(0, \sigma^2_{\xi, f}) \) [10], while small-scale fading follows Nakagami-
\( m \) distribution, so that \( |h^f_{\xi,k} \rightarrow x_j|^2 \sim \Gamma(m, 1/m). \)

Let \( \mathcal{K} := \left\{ \mathcal{K}_k \right\}_{k=1}^K \) and \( \{\mathcal{E}_k\}_{k=1}^K \) denote the set of locations where the \( K \) PU transmitters reside and the PU transmit-
\( \) powers, respectively, which are acquired via sensing [8], [9]. The CR network employs a distributed scheduling algorithm, such as C-MAC [11] to prevent mutual interference. Then, the
\( \) instantaneous signal-to-interference-plus-noise ratio (SINR) of link \( (i,j) \) on band \( f \in \mathcal{F} \), is given by
\[
\gamma^f_{ij} := \frac{p^f_{ij} g^f_{\xi,k} \rightarrow x_j}{\sigma^2_{N,j} + \sum_{k \in \mathcal{K}} p^f_{ki} g^f_{\xi,k} \rightarrow x_j}
\]
where \( \sigma^2_{N,j} \) is the receiver noise variance. Instantaneous values of \( g^f_{\xi,k} \rightarrow x_j \) and \( g^f_{\xi,k} \rightarrow x_j \) are assumed unavailable at node \( i \), due to fast fading. The PU-to-CR channels are, in
\( \) general, also challenging to acquire due to lack of inter-system cooperation [4]. Given the uncertainty in the SINR values at
\( \) node \( i \), the notion of outage probability is used to quantify
\( \) network rates and link-reliabilities [6]. A packet transmitted at node \( i \) is lost and outage is declared at node \( j \) if \( \gamma^f_{ij} \leq \bar{\gamma} \), where the SINR threshold \( \bar{\gamma} \) depends on the receiver structure, modulation, and bit error rate (BER) requirements [12]. Then, the outage probability on link \( (i,j) \) and band \( f \) is given by
\[
O^f_{ij}(p^f_{ij}) := \Pr \left\{ \gamma^f_{ij} \leq \bar{\gamma} \right\} \quad (i,j) \in \mathcal{E}, f \in \mathcal{F}.
\]
QoS requirements of the network traffic limit the outage probability of the links, i.e., \( O^f_{ij}(p^f_{ij}) \leq c_{ij} \) for all \( (i,j) \in \mathcal{E} \), \( f \in \mathcal{F} \). Let \( \pi^f_{ij} \) be the fraction of time allocated by the MAC
\( \) layer to link \( (i,j) \) and band \( f \). These fractions are fixed for a given scheduling algorithm, and can be calculated a priori; see e.g., [11]. Then, the outage probabilities can be used to define the average MAC-layer capacity of link \( (i,j) \) as
\[
c_{ij} := \sum_{f \in \mathcal{F}} \pi^f_{ij} (1 - O^f_{ij}(p^f_{ij}))
\]
where \( \pi_{ij} \) is the PHY layer transmission rate at node \( i \) and band \( f \).

Consider a PU receiver located at \( u^r \), and let \( \tau^f_r \) denote the total interference power that is inflicted by the CR nodes to that node on band \( f \). Let \( \mathcal{E}_\tau \) be all possible subsets \( \mathcal{E} \in \mathcal{E} \) of links that can be scheduled simultaneously by the MAC layer. For given channel realizations \( \{g^f_{\xi,k} \rightarrow u^r\} \in \mathcal{N} \), the maximum interference suffered by the PU receiver is then
\[
\tau^f_r = \max_{\mathcal{E} \in \mathcal{E}_\tau} \sum_{(i,j) \in \mathcal{E}} p^f_{ij} g^f_{\xi,k} \rightarrow u^r, \quad f \in \mathcal{F}.
\]
In most cases, the number \( L \) of subsets may be too large to enumerate, and the following simple upper bound on the received interference may be used instead
\[
\tau^f_r = \sum_{(i,j) \in \mathcal{E}} p^f_{ij} g^f_{\xi,k} \rightarrow u^r, \quad f \in \mathcal{F}.
\]
Alternatively, observe that at any time, the transmit power of \( i \) is at most \( \sum_{j \in \mathcal{E}} \), which yields
\[
\tau^f_r = \max_{(i,j) \in \mathcal{E}} \left[ \sum_{(i,j) \in \mathcal{E}} p^f_{ij} g^f_{\xi,k} \rightarrow u^r, \quad f \in \mathcal{F} \right].
\]
Finally, to ensure PU protection, the following average interference constraint is enforced \( \mathbb{E}\left[\tau^f_r \right] \leq \tau_{\text{max}} \) for all \( f \in \mathcal{F} \), where \( \tau_{\text{max}} \) is the maximum interference power that can be tolerated by the PU receiver [13].

The caveat in applying the maximum interference constraint is that estimating the PU location \( u^r \) precisely may be challenging without explicit cooperation with the PU receivers. To overcome this hurdle, constraints should be imposed for all \( u^r \in \cup_{k \in K} S_k \), where \( S_k \) denotes the coverage region of PU transmitter \( k \), and can be estimated as shown in e.g., [14], [15]. Henceforth, the entire set of known or potential PU receiver
\( \) locations is denoted by \( S := \cup_{k \in K} S_k \).

B. Network Layer and Above

The CR network supports multiple data sessions \( m \in \mathcal{M} \), and the optimization variables at the network and transport
\( \) layers are the session rates \( \{R(m)\} \) and link rates \( \{z^f_{ij}(m)\} \). The region of achievable rates depends on the type of routing or network coding protocol used. When a fixed-path routing strategy is employed, each session \( m \in \mathcal{M} \) is associated with a pre-specified set of links \( \mathcal{E}(m) \subseteq \mathcal{E} \) over which packets are persistently routed; this yields the constraints
\[
\sum_{m \in \mathcal{M}} R(m) \leq c_{ij} \quad (i,j) \in \mathcal{E}
\]
with the link rates \( z_{ij}(m) = R(m) \) for all \( (i,j) \in \mathcal{E}(m) \). On the other hand, when using network coding, each session supports a multicast session \( m \in \mathcal{M} \) from source node \( s(m) \) to the set of sink nodes \( T(m) \). In this case, the network-layer capacity region can be described by introducing auxiliary variables \( \{z^f_{ij}(m)\} \), with \( z_{ij}(m,t) \geq 0 \) representing the virtual transmission rate (also called virtual flow) on link \( (i,j) \in \mathcal{E} \) for sink \( t \in T(m) \), and session \( m \in \mathcal{M} \). Virtual and real flows are related through the following set of constraints [2]:
\[
\sum_{j \in \mathcal{E}(m)} r^f_{ij}(m,t) - \sum_{j \in \mathcal{E}} r^f_{j}(m,t) = R(m) \mathbb{1}_{i = s(m)}
\]
\forall (i,j) \in \mathcal{E}, t \in T(m), m \in \mathcal{M}
\]
\[
\sum_{t \in T(m)} z^f_{ij}(m,t) \leq z^f_{ij}(m), \quad \forall (i,j) \in \mathcal{E}, t \in T(m), m \in \mathcal{M}
\]
\[
\sum_{m \in \mathcal{M}} z^f_{ij}(m) \leq c_{ij}, \quad \forall (i,j) \in \mathcal{E}, m \in \mathcal{M}
\]
where \( \mathbb{1}_{\{\cdot\}} \) is the indicator function that takes the value 1 if its argument is true, and zero otherwise. Multopath routing is
a special case of (9) with only one sink per session; that is, \(|T^{(m)}| = 1\). In any of the aforementioned cases, the rate region can be succinctly denoted as \((R^{(m)}, \zeta^{(m)}) \in \Lambda\), where the polytope \(\Lambda\) accounts for the linear constraints corresponding to the specific protocol adopted by the CRs.

C. Optimal Rate and Power Allocation

The CR network aims to maximize the session rates \(\{R^{(m)}\}\) while spending minimum power \(\{p^{(m)}_{ij}\}\) per node. Towards this end, consider concave utility \(U^{(m)}(R^{(m)})\) and convex cost \(C^{(m)}_{ij}(p^{(m)}_{ij})\) functions to balance session rate and power requirements. Then, optimal rate and power allocation can be performed by solving the following optimization problem

\[
\begin{align*}
\text{(P)}: \quad & \max_{\{p^{(m)}_{ij} \geq 0\}, \{z^{(m)}_{ij} \geq 0\}, \{R^{(m)} \geq 0\}} \sum_{m \in M} U^{(m)}(R^{(m)}) - \sum_{(i,j) \in E} \sum_{f \in F} C^{(m)}_{ij}(p^{(m)}_{ij}) \\
\text{s.t. (R\text{--}m)_j, z^{(m)}_{ij} \in \Lambda,} & \\
\sum_{m \in M} z^{(m)}_{ij} & \leq \sum_{f \in F} C^{(m)}_{ij}(p^{(m)}_{ij}) (1 - O^{(m)}_{ij}(p^{(m)}_{ij})) (i,j) \in E, \forall m \in M \quad (10a) \\
E[p^{(m)}_{ij}] & \leq \tilde{\pi}^{\max}, \quad f \in F, \forall u' \in S \quad (10b) \\
p^{(m)}_{ij} & \leq \tilde{\pi}^{\max}, \quad f \in F \quad (10c) \\
O^{(m)}_{ij}(p^{(m)}_{ij}) & \leq \epsilon, \quad (i,j) \in E, f \in F. \quad (10d)
\end{align*}
\]

Solving \((\text{P})\) is challenging primarily because of the constraints (10b) and (10e), as they involve specifying an expression for \(O^{(m)}_{ij}(p^{(m)}_{ij})\), which could be intractable for realistic channel distributions. In the next section, an approximate convex counterpart of \((\text{P})\) will be derived, which can then be solved using efficient methods. Before concluding this section, some remarks are in order.

Remark 1. Severe interference from PU transmitters may render it impossible for some links to meet the maximum outage constraint (10e), even when transmitting at maximum power \(p^{\max}\). However, in practice, such links will never be scheduled by the MAC layer, as even control signals from neighboring nodes will be lost due to outage events; i.e., \(\pi^{f}_{ij} = 0\). To avoid \((\text{P})\) from being infeasible, variables \(p^{(m)}_{ij}\) for which \(\pi^{f}_{ij} = 0\) should be eliminated from the problem.

Remark 2. Problem \((\text{P})\) needs to be (re-)solved only when SINR or interference statistics vary. Thus, operational parameters \(\{p^{(m)}_{ij}\}, \{R^{(m)}\}, \{z^{(m)}_{ij}\}\) are updated whenever the sensing module detects a change in the channel statistics, PU transmit-powers, or locations.

III. APPROXIMATE CONVEX FORMULATION

To formulate a convex problem approximating \((\text{P})\), observe first that except for (10b), (10c), and (10e), all other constraints are linear in \(p^{(m)}_{ij}, R_{m}\) and \(z^{(m)}_{ij}\). Next, consider the interference constraint (10c), which can equivalently be rewritten as

\[
\sup_{u' \in S} E[\pi^{f}_{ij}] \leq \tilde{\pi}^{\max} \quad f \in F. \quad (11)
\]

When adopting (6) as the expression for \(\psi^{\ell}_{ij}\), (11) becomes

\[
\sup_{u' \in S} \sum_{(i,j) \in E} p^{\ell}_{ij} g^{\ell}_{x_{ij} \rightarrow u'} \leq \tilde{\pi}^{\max}. \quad (12a)
\]

where \(g^{\ell}_{x_{ij} \rightarrow u'} := E[g^{\ell}_{x_{ij} \rightarrow u'}]\). The left-hand side of (12a) is convex in the optimization variables \(\{p^{\ell}_{ij}\}\), since it is a supremum of affine (hence, convex) functions. Notice, though, that \(g^{\ell}_{x_{ij} \rightarrow u'}\) is proportional to \(|x_{ij} - u'|^{\eta}\), and it is therefore non-convex in \(u'\). Similarly, when using the expression in (7), the expectation operator can be taken inside the summation, thus yielding the constraint

\[
\sup_{u' \in S} \sum_{(i,j) \in E} \max_{f \in F} p^{\ell}_{ij} g^{\ell}_{x_{ij} \rightarrow u'} \leq \tilde{\pi}^{\max}. \quad (12b)
\]

The left-hand side of (12b) is again a convex function in \(\{p^{\ell}_{ij}\}\) since the expression after the supremum operator is a convex function of \(\{p^{\ell}_{ij}\}\). Using similar arguments, it can again be shown that (5) yields the convex constraint per subset \(E_{\ell}^{f}\)

\[
\sup_{x' \in S} \sum_{(i,j) \in E_{\ell}^{f}} p^{\ell}_{ij} g^{\ell}_{x_{ij} \rightarrow u'} \leq \tilde{\pi}^{\max}. \quad (12c)
\]

For the constraint (10e), notice that \(O^{(m)}_{ij}(p^{(m)}_{ij})\) is always a monotonically decreasing function of \(p^{(m)}_{ij}\). This means that (10e) is equivalent simply to a lower bound constraint on \(p^{(m)}_{ij}\). An expression for \(O^{(m)}_{ij}(p^{(m)}_{ij})\) is in general difficult to evaluate, and next two approximations are introduced to simplify (10e) and (10b).

The channel \(g^{\ell}_{x_{ij} \rightarrow u'}\) collects the effects of shadowing and Nakagami-\(m\) fading, and follows a Gamma-lognormal distribution. This distribution is in general well-approximated by the log-normal distribution with parameters [10, Ch. 2]

\[
\begin{align*}
\mu^{f}_{ij} & = \kappa^{-1}(\psi(0, m) - \ln(m)) - \kappa(\eta \ln \|x_{ij} - x_{j}\|_2 + \ln g_{ij}) \quad (13a) \\
\sigma^{2}_{ij,f} & = \kappa^{-2}(1, m) + \sigma^{2}_{f} \quad (13b)
\end{align*}
\]

where \(\psi(\cdot, \cdot)\) is the polygamma function, and \(\kappa := \ln(10)/10\). This also allows the average CR-to-PU channel gains \(g^{\ell}_{x_{ij} \rightarrow u'}\) to be re-expressed in terms of the known moments (13a)-(13b) as \(g^{\ell}_{x_{ij} \rightarrow u'} = \exp(\kappa p^{f}_{ij} + 2 \sigma^{2}_{f})\). Next, (2) can be rewritten as

\[
\frac{p^{f}_{ij}}{\gamma^{f}_{ij}} = \sigma^{2}_{X_{ij}} \left(\tilde{g}^{\ell}_{x_{ij} \rightarrow x_{j}}\right)^{-1} + \sum_{k \in K} \tilde{p}_{k} \left(\tilde{g}^{\ell}_{x_{k} \rightarrow x_{j}}\right)^{-1} g^{\ell}_{u_{k} \rightarrow x_{j}}. \quad (14)
\]

Recalling that inverse and product of log-normal random variables also follow a log-normal distribution, it follows that the right-hand side of (14) is a sum of log-normal random variables. Since no closed-form expression for the distribution function of a sum of log-normal random variables is available, a second layer of approximation needs to be performed. A popular class of techniques in this regard involve approximating the sum of log-normal random variables with a single log-normal random variable; see e.g., [6], [7], [16], [17], and references therein. Using one of these techniques to
calculate the mean $\tilde{\mu}_{ij}^f$ and variance $(\tilde{\sigma}_{ij}^f)^2$ of the resulting distribution, it follows that

$$10\log_{10}\left( p_{ij}^f / \gamma_{ij} \right) \sim N(\tilde{\mu}_{ij}^f, (\tilde{\sigma}_{ij}^f)^2).$$

(15)

Using approximation (15), the outage probability of link $(i, j)$ is given by

$$O_{ij}^f(p_{ij}^f) \approx 1 - Q\left( \frac{\kappa \ln(\bar{\gamma}) - \kappa \ln(p_{ij}^f) + \tilde{\mu}_{ij}^f}{\tilde{\sigma}_{ij}^f} \right).$$

(16)

where $Q(\cdot)$ is the Gaussian tail function, and which yields

$$\sum_{m \in M} z_{ij}^{(m)} \leq \sum_{f \in F} c_{ij}^f \pi_{ij}^f Q\left( \frac{\kappa \ln(\bar{\gamma}) - \kappa \ln(p_{ij}^f) - \tilde{\mu}_{ij}^f}{\tilde{\sigma}_{ij}^f} \right).$$

(17)

Note however, that the right-hand side of (17) is still not a concave function of $p_{ij}^f$. Interestingly though, the constraint region specified by (17) and (10e) combined, is convex for $\epsilon \leq 1/2$. To show this, notice that the right-hand side is concave in the sub-region where $1 - O_{ij}^f(p_{ij}^f) \geq Q\left( \frac{\kappa \ln(\bar{\gamma})}{\tilde{\sigma}_{ij}^f} \right)$. Since $Q(x)$ is at most 1/2 for $x \geq 0$, it follows that (17) is indeed a convex constraint if the maximum allowable outage probability $\epsilon \leq 1/2$. Note that for realistic values of the parameters, $\epsilon$ is typically allowed to be as high as 0.98. In summary, (P0) can be solved using efficient methods by replacing (10c) with (12), (10b) with (17), and (10e) with a lower bound on $p_{ij}^f$.

IV. NETWORK CODING WITH BROADCAST

If each transmission from node $i$ can be decoded by all its neighbors, the use of network coding may yield even higher session rates. For simplicity, consider a single multicast session with session rate $R$. The capacity region in this case is again described by the virtual flow variables $\{\tilde{\gamma}_{ij}^f\}$, which abide by the flow conservation constraints (9a). At the PHY layer however, the power allocation variable at node $i$ is simply the broadcast power $p_{ij}^f$, rather than the point-to-point power allocations $p_{ij}^f$. Therefore, the received SINR at node $j$ is modified as

$$\tilde{\gamma}_{ij}^f = \frac{p_{ij}^f g_{ij}^f - \sigma_u^2}{\sigma_u^2 N + \sum_{k \in K^f} p_k^f g_{kj}^f - \sigma_u^2}.$$  

(18)

Since coded packets are now broadcast, the notion of outage event has to be properly extended to account for the successful packet reception at multiple nodes. Specifically, the relationship between the SINRs and virtual flow variables is now given by

$$\sum_{j \in K} \tilde{\gamma}_{ij}^f \leq \sum_{f \in F} c_{ij}^f \pi_{ij}^f \Pr\left( \bigcup_{j \in K} \tilde{\gamma}_{ij}^f \geq \tilde{\gamma} \right) \quad K \subset N(i)$$

(19)

where $\pi_{ij}^f$ is the fraction of time node $i$ is scheduled for transmission on band $f$ [11], [18], and $N(i)$ is the set of neighbors of node $i$, i.e., $N(i) = \{ j : (i,j) \in E \}$. Finally, the interference received at PU nodes can be bounded as

$$\sup_{u^* \in S} \left[ p_{ij}^f g_{ij}^f - u^* \right] \leq \epsilon_{\text{max}}, \quad i \in N, f \in F.$$  

(20)

Observe that except for (19), most constraints are similar in form to those in (P0), and are therefore convex. The right-hand side of (19) is however intractable, especially because not even approximate expressions are available for the joint distribution function required here. If however, the shadow fading gains and consequently the SINRs $\tilde{\gamma}_{ij}^f$ are assumed uncorrelated, the following expression can be obtained by applying the two approximations outlined in Section III

$$\Pr\left( \bigcup_{j \in K} \tilde{\gamma}_{ij}^f \geq \tilde{\gamma} \right)$$

$$= 1 - \prod_{j \in K} \left( 1 - Q\left( \frac{\kappa \ln(\bar{\gamma}) - \kappa \ln(p_{ij}^f) - \tilde{\mu}_{ij}^f}{\tilde{\sigma}_{ij}^f} \right) \right).$$

(21)

As in Section III, it is possible to establish concavity of the right-hand side of (21), for $\epsilon \leq 1/2$. Note that each multiplicant in (21), henceforth denoted by $f_j(p_{ij}^f)$, is a convex, decreasing function that lies between 0 and $1 - \epsilon$. The second derivative of the product of functions $f_j(p_{ij}^f)$ is given by

$$f_j(p_{ij}^f) = \frac{\sum_{j \neq j_1, j_2} f_j(p_{ij}^f) f_{j_1}(p_{ij}^f) f_{j_2}(p_{ij}^f) \Pi_{j \neq j_1, j_2} f_j(p_{ij}^f) + \sum_{j_1, j_2} f_{j_1}(p_{ij}^f) f_{j_2}(p_{ij}^f) f_j(p_{ij}^f)}{f_j(p_{ij}^f)}. \quad \text{The first term is positive because} \quad f_j(p_{ij}^f) \leq 0 \text{ for all } j_1 \in K, \text{ while the second term is positive because} \quad f_j(p_{ij}^f) \geq 0 \text{ for all } j_1 \in K. \text{ Thus, the product is also convex, which implies that the expression on the right of (21) is concave.}

V. NUMERICAL TESTS

Consider the scenario depicted in Fig. 1, involving a PU node sharing a band with a CR network comprising 5 nodes. To better appreciate the different aspects of the optimization problem (P0), only a single frequency band is considered. The PU node transmits at power $\tilde{p}_k = 1$ W. The channel gain parameters are set to their typical values in urban environments [10], with $g_i = 1$, $\eta = 4$, $\sigma_{s,f} = 8$ dB, and $m = 2$. Shadowing is spatially correlated, with coherence distance 10 m. The CRs transmit at a power of at most $p_{\text{max}} = 1$ W. The PU transmitter is assumed to have a coverage region given by a disk of radius 10 m, where potential PU receivers may
lie. Packet transmissions among CR nodes are successful if the SINR exceeds 10 dB, and the maximum allowed outage probability is set to $\epsilon = 0.5$. The MAC scheduler ensures that all links are accessed with equal probability; i.e., $\pi_{ij} = 0.1$ for all $(i, j) \in \mathcal{E}$. For convenience, the transmission rate $c_i$ is set to 10 packets per unit time, thus making the link-layer rates belong to $[0, 1]$. The network supports two sessions, one from node 1 to node 5, and another in the reverse direction, with utility functions $U(m)(R(m)) = \log(R(m))$, $m = 1, 2$.

Constraint (12b) is utilized to limit the interference inflicted to primary devices within the PU coverage region. Although the constraint region is in this case convex, the supremum over $u^* \in \mathcal{S}$ must still be evaluated for given values of $\{p^f_{i,j}\}$. However, since the vector $u^*$ is only two-dimensional, a simple grid search is used to obtain the result with low complexity. Fig. 2 shows the power allocations for different values of tolerable thresholds $\nu_{ij}$ at the PU receivers. The link color and thickness are proportional to the allocated power. With $\nu_{ij} = -50$ dBW, the interference inflicted to the region $\mathcal{S}$ is the limiting factor, especially for node 3. This forces the the longer path 1 $\leftrightarrow$ 2 $\leftrightarrow$ 4 $\leftrightarrow$ 5 to carry a large percentage of the total end-to-end traffic. If the interference constraint is eased to $\nu_{ij} = -40$ dBW however, node 3 is able to route a significant amount of extra traffic, and increase system throughput.

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