Lesson objective: to recognize probabilities in a simple game of chance and to become familiar with the idea of a fair game.

Materials:

- Two-color spinners for the entire class (one for every two students)
- Labsheet 1.2

## Launch

Tell students that we will be analyzing the spinner game to decide if it is fair or not.

My brother invented a game, and he wants to know if it is fair.

The Spinner Game: (explain while showing the students the spinner)
Two players play with a two-color spinner. The spinner should have equal probability of landing on each color (in our case blue and yellow). The players much decide who will be player A and who will be player B. A turn consists of two spins. Each turn will either be matching spins or mismatching spins. Player A scores 1 point for every turn which is a match. Player B scores 2 points for every turn which is a mismatch. My brother decided that player A should only get one point for every match, since there are two different matches (yellow/yellow and blue/blue) but only one mismatch (one of each).

Is my brother's spinner game fair? Why? Why not? What makes a game fair?

Make sure your spins are random. What kind of things could bias the results? (slanted desktops, starting position of spinner, creases in paper, etc)

## Explore

Hand out Spinners and Labsheet 1.2. Students work in pairs and play The Spinner Game.

Write example of scoring on the board:

| Turn Number | Result | Player A Score | Player B Score |
|---|---|---|---|
| 1 | yellow/yellow | 1 | |
| 2 | yellow/blue | | 2 |

Each pair fills out one lab sheet to record data and find experimental and theoretical possibilities.

## Summarize

Write down the winner (player A or player B) for every group in the class.
Discuss the answers to questions as a class.

1. *answers will vary*

2. List all of the different possible outcomes for a turn in this game. Write each outcome in the form *color of first spin/color of second spin*.
*____blue/yellow____blue/blue___yellow/yellow___yellow/blue_____*

3. What are the theoretical probabilities of a match and a mismatch?

$$P(match) = \frac{\text{number of outcomes that are matches}}{\text{number of possible outcomes}} = \frac{2}{4} = \frac{1}{2}$$

$$P(mismatch) = \frac{\text{number of outcomes that are mismatches}}{\text{number of possible outcomes}} = \frac{2}{4} = \frac{1}{2}$$

4. Compare the experimental and theoretical possibilities. Is this game fair?
*The experimental possibilities will be reasonably close to the theoretical probabilities. Show that P(match) and P(mismatch) are approximately equal, and recall the faulty assumption that led to an unfair scoring system. (The game is not fair).*

Repeat definition of a fair game: "a game is fair if each player has the same expected score for a given number of turns."

Extensions:

- What can we change about this game to make it fair?
*Change the scoring to make matches and mismatches equally valuable (since they are equally probable).*

- What if we played the same game using a spinner with three equally likely colors? Is that game fair? What would be a fair way to score that game?
*Not fair; P(match)=1/3, P(mismatch)=2/3. Would be fair if A got 2 for a match and B got 1 for a mismatch.*

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Notes:
Things to change

Things to skip

Things to be careful of