AEM4301 final project report

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Abstract

The mission to be considered for this project is the optimal flyby of Venus from Earth that creates the largest final heliocentric speed. The method of patched conics will be used to determine the trajectory which is optimal to the mission. The final trajectory of the spacecraft after leaving Venus will be an ellipse and the semi-major axis of this final ellipse will be determined as well as the necessary initial phase angle of Venus with respect to Earth at the time of burn from Low Earth Orbit.

1 Problem Statement

The goal of the mission is to find a trajectory from the Earth to Venus that will elicit an optimal flyby. The conditions for an optimal flyby are such that the ratio of final heliocentric speed to that of the change in velocity required for the spacecraft to leave Low Earth Orbit will be the greatest. This corresponds to the greatest possible post-flyby speed of the spacecraft with the least amount of required fuel.

After determining the parameters of the optimal trajectory, it will be necessary to find the phase angle between Earth and Venus at time of burn as well as the semi-major axis of the spacecraft's final heliocentric orbit.

2 Methodology

The spacecraft's mission trajectory was determined using the method of patched conics. There are four trajectories considered in this method. The first is the hyperbola as the spacecraft leaves its parking orbit of 200km. The second is the transfer ellipse. The third is the hyperbola as the spacecraft completes its flyby. The fourth is the final orbit of the spacecraft about the Sun. However, the transfer ellipse must be considered first in this method because it is the independent variable of the problem.

See section 3 for the valid range of semi-major axis values for the transfer ellipse.

2.1 The transfer ellipse

For a given semi-major axis distance, the important elements of the transfer orbit can be obtained, each one proceeding from the previous.

$$E_{transfer} = \frac{-\mu_s}{2a} \tag{1}$$

$$V_{apoapsis} = \sqrt{2(E_{transfer} + \frac{\mu_s}{r_a})} \tag{2}$$

$$h_{transfer} = V_{apoapsis} r_a \tag{3}$$

Because the intersection of the spacecraft's orbit with that of Venus always occurs at the same case, when the radius of the transfer ellipse equals the that of of Venus's orbit, it is possible to calculate both the $V_1^{(v)}$ and γ, the flight path angle, at intersection.

$$V_1^{(v)} = \sqrt{2(E_{transfer} + \frac{\mu_s}{r_v})} \tag{4}$$

$$\alpha_1 = \arccos \frac{h_{transfer}}{V_1^{(v)} r_v} \tag{5}$$

Once $V_1^{(v)}$ and γ are determined, the method of patched conics for the particular case of semi-major axis length may be continued in terms of the flyby of Venus itself.

2.2 Venus flyby

Knowing the heliocentric speed of Venus and the velocity of the spacecraft on the transfer ellipse at intercept, the hyperbolic excess speed, $V_{\infty 1}$, as the spacecraft enters the sphere of influence can be determined. V_V is the velocity of Venus on its orbit. The flight path angle, γ, on the transfer ellipse is equal to the angle α_1 (see figure 1).

$$V_{\infty 1} = \sqrt{(V_1^{(v)})^2 + V_V^2 - V_1^{(v)} V_V \cos \alpha_1} \tag{6}$$

ϕ_1, the angle between the velocity of Venus and the hyperbolic excess speed, can be found from the Law of Sines.

$$\phi_1 = \arcsin\left(\frac{V_1^{(v)}}{V_{\infty 1}} \sin\alpha_1\right) \tag{7}$$

Next, determine the elements of the flyby hyperbola.

$$E_{flyby} = \frac{(V_{\infty 1})^2}{2} \tag{8}$$

$$h_{flyby} = R_{venus}\sqrt{(V_{\infty 1})^2 + \frac{2\mu_v}{R_{venus}}} \tag{9}$$

$$e_{flyby} = \sqrt{1 + \frac{2E_{flyby}(h_{flyby})^2}{\mu_v^2)}} \tag{10}$$

At this point the turning angle can be determined. The turning angle,δ, is the angle between the hyperbolic excess velocities entering and leaving Venus's sphere of influence.

$$\delta = 2\sin\frac{1}{e_{flyby}} \tag{11}$$

The flyby can occur either on the sunlit side or the dark side of the planet. The angle between the velocity vector of the planet and the post flyby heliocentric velocity vector, ϕ_2, is dependent on this choice.

$$\phi_2 = \phi_1 \mp \delta \tag{12}$$

For a sunlit flyby, the minus sign is used. For a darkside flyby, the plus sign is used. Once ϕ_2 is determined, the magnitude of the post-flyby heliocentric velocity is given by the Law of Cosines. The magnitudes of $V_{\infty 1}$ and $V_{\infty 2}$ are the same (even though the directions are different).

$$V_2^{(v)} = \sqrt{V_V^2 + V_{\infty 2}^2 - 2V_V V_{\infty 2}\cos(\pi - \phi_2)} \tag{13}$$

The determination of α_2 is inconsequential to this analysis.

2.3 Leaving Low Earth Orbit

Because we're not only concerned with the final heliocentric speed of the satellite but how it compares with the change in velocity required to leave the Earth's sphere of influence, we must consider the hyperbola as the spacecraft leaves Low Earth Orbit. This ΔV can be derived from the following equations based on what has already been found in this analysis.

$$V_\infty = V_{Earth/Sun} - V_{apoapsis(transfer)} \tag{14}$$

$$V_{LEO} = \sqrt{\frac{\mu_E}{r_{LEO}}} \tag{15}$$

$$\Delta V = V_{LEO}\sqrt{2 + \frac{V_\infty^2}{V_{LEO}^2}} - 1 \tag{16}$$

2.4 Post-flyby trajectory

Once the optimal $V_2^{(v)}$ is determined from the graphs of figures 5 and 4, the semi-major axis of the final heliocentric orbit can be derived. Equation 17 is a form of the energy equation.

$$a_{final} = \frac{-\mu_s}{2(\frac{(V_2^V)^2}{2} - \frac{\mu_s}{r_v})} \tag{17}$$

2.5 Phase angles at launch

The phase angle is defined as the angle between the departure planet and the arrival planet at the time of burn. A phase angle exists because each planet has a different rotational rate about the Sun, so an the offset must be determined to compensate for the fact that the transfer ellipse has a time of flight associated with it. Finding the phase angle is made slightly trickier considering that this is a non-Hohmann transfer. After finding the true anomaly of the spacecraft on the transfer ellipse at intercept, the true anomaly of Venus at the point of burn was found. Earth is at apoapsis on the transfer ellipse at the time of burn, so knowing that, it is possible to find the angle between Earth and Venus at burn, Φ

3 Discussion

Transfer ellipse There is a well-defined range of semi-major axis values for the transfer ellipse from Earth's orbit to Venus's orbit. This is because the transfer orbit must both intersect with the Venusian orbit *and* retain the Sun as a focus. Therefore, the transfer ellipse has valid periapsis distances that range from zero (at the Sun—a rather asymptotic value) to the radius of Venus's orbit. Furthermore, the apoapsis distance for any valid transfer orbit is constant: the radius of Earth's orbit around the Sun.

A script was developed in MATLAB that would simulate missions for a valid range of the transfer ellipse's semi-major axis ranging from 74.8×10^6 km to 128.9×10^6 km.

Planetary flyby The aiming radius was chosen to be the impact parameter. In reality this would cause the spacecraft to collide with the planet's atmosphere or the planet itself; however, for the sake of approximation, flying at "treetop" level gives the optimum trajectory.

For the flyby hyperbola one has the choice of a sunlit or dark side flyby. From figure 4 we can see that the final heliocentric speed of the spacecraft is greater in each case for a sunlit side flyby than for a dark side flyby. Figure 5 shows the ratio of the final heliocentric velocity to the required delta V. Again, the sunlit side flyby is always a better choice.

As the transfer orbit's semi-major axis increases, the final heliocentric velocity decreases exponentially. However, the ratio of $V_2^{(v)}$ to ΔV is greatest near the longest possible distance of semi-major axis (figure 5).

An interesting thing to note about figures 5 and 4 is that a discontinuity occurs when the transfer semi-major axis is 116.9×10^6 km. In this case, the spacecraft intersects with Venus's orbit at a true anomaly of $3\pi/2$ (the "top" of Venus's orbit).

Phase angles θ_f is the angle from periapsis of the transfer ellipse to the point of intersection between the spacecraft and the orbit of Venus. Since the orbit of Venus is assumed to be circular, a circular rotational rate was derived (ω). From this parameter and the time of flight, t, θ_i can be determined from equation 18.

$$\theta_i = \theta_f - \omega t \tag{18}$$

Using universal variables, the time of flight, t, was calculated to be 7.71×10^6 s and θ_f at intersection was determined to be 274.160°. ω is found using equation 19.

$$\omega = \sqrt{\frac{\mu_s}{a}} \tag{19}$$

This gives an θ_i of 2.2886 radians (131.1°). Because the burn occurs at apoapsis on the transfer ellipse, we know that Φ is the angle with respect to the line of apses through both orbits. This then causes Φ to be the angle in figure 3 and equation 20.

$$\Phi = 180° - \theta_i \tag{20}$$

4 Conclusions

Considering the graph of figure 5, the optimal ratio of $V_2^{(v)}$ to ΔV from Low Earth Orbit occurs with a sunlit flyby of Venus and a transfer ellipse with a semi-major axis of 122.2×10^6km. According to figure 4, the $V_2^{(v)}$ that corresponds to this semi-major axis length is 42km/s.

Equations 17 and 20 give the desired semi-major axis of the optimal post-flyby orbit as well as the phase angle required at launch for this trajectory.

$$a_{final} = 192.6 \times 10^6 \text{ km}$$
$$\Phi = 48.8°$$

A Figures

Figure 1: Approach vectors

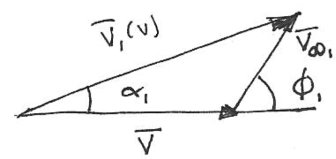

Figure 2: Departure vectors

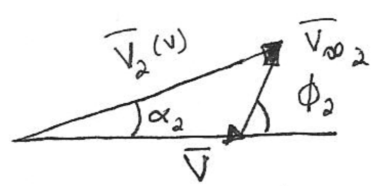

Figure 3: Phase angle at launch

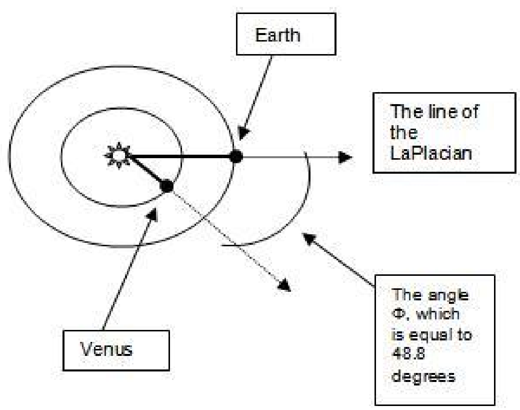

Figure 4: Final heliocentric velocity versus varying values of the semi-major axis of the transfer ellipse

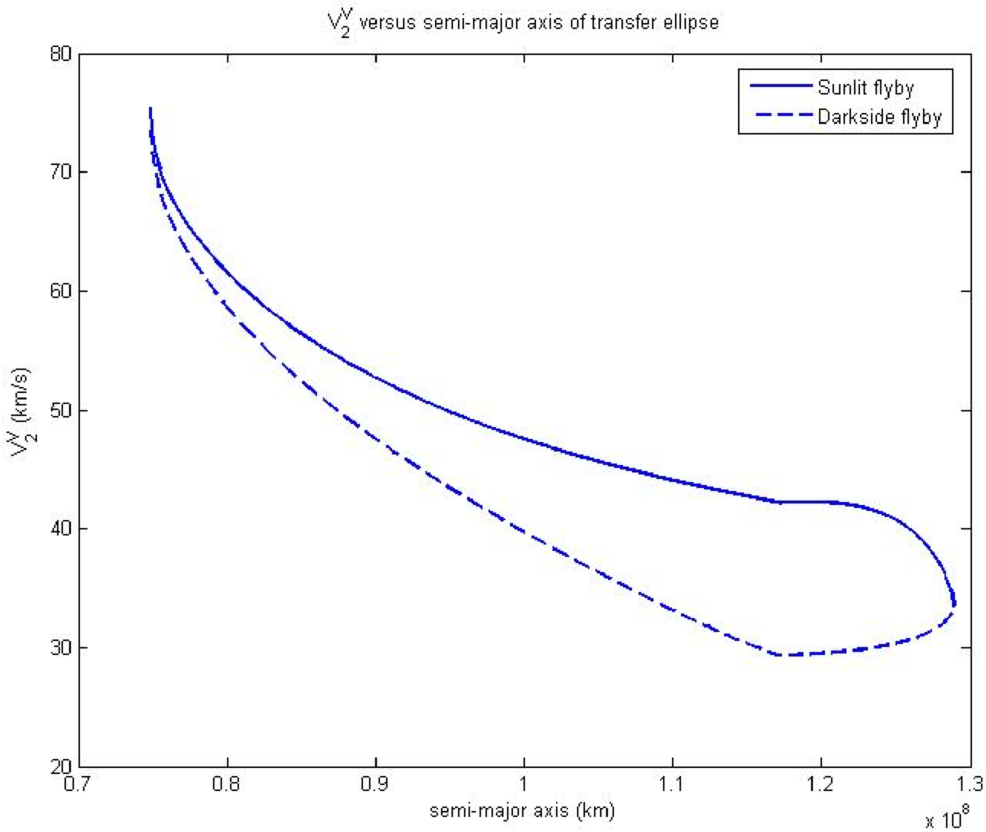

Figure 5: The ratio of final heliocentric velocity to delta-v from Low Earth Orbit