BMEn 5351
Cell Engineering
Spring 2007
Homework #1: Basic Physical and Chemical Principles

Due January 25, 2007. Work individually. Turn in on 8.5 x 11 in. paper (clean edges) with your name on the upper right corner along with the course and assignment numbers. Place a single staple in the upper left corner. Number pages sequentially.

1. If a protein is synthesized in the cell body of a neuron, it must diffuse over a long distance down the axon in order to reach the synapse. Note that axons can be up to 1 m (10^6 μm) long! Assuming the protein has a diffusion coefficient of 5 $\mu m^2/s$, how long will it take it to diffuse from the cell body to a synapse 500 μm away? Since the axon is a long, thin extension, the diffusion process can be regarded as one-dimensional.
2. Nuclear pores control the transport of material into and out of the nucleus (we will review the mechanisms in the upcoming Smith paper). Use Fick's Law to calculate the flux of a protein that has a cytoplasmic concentration of 6 μM and a nucleoplasmic concentration of 3 μM (in units of molecules/m^2 s; assume D=15 $\mu m^2/s$, and that the nuclear membrane is 30 nm thick). Assuming that the cross-sectional area of the pore is 20 nm^2, calculate the net number of molecules per second that pass from the cytoplasm to the nucleoplasm per second.
3. Animal cells have evolved mechanisms for active crawling, which enables them to travel at speeds of ~1-10 μm/min. How much more effective is this than diffusion? Use the Stokes-Einstein relation to calculate the diffusion coefficient for an animal cell having radius 7 μm, and that is diffusing in water at 37^{o}C (viscosity is 0.0007 kg/m s). Then use this value to calculate the root-mean-squared (r.m.s.) displacement of the cell in three-dimensional space over 1 day.
4. For the reversible chemical reaction A<->B, use the following values for ΔG^o to then calculate the ratio of the molar concentration of B to the molar concentration of A at equilibrium: 0, -1, -2, -5, -10, -20 k_BT/molecule (note that hydrogen bonds are ~-1.6 k_BT/bond).
5. For the reversible association reaction A + B <-> AB, which has a K_D=4 μM, calculate the ΔG^o. In the cell, reactions are hardly ever at equilibrium. If [A]=20 μM, [B]=3 μM, and [AB]=1 μM, then calculate ΔG. Will the reaction proceed forward or reverse under these conditions?
6. Reactions in the cell are couple to many other reactions, so that they form a large, interconnected web of reactions. Consider the coupling of two reactions and write the equations for the time rate of change of the molar concentrations of each of the four species in this mini-network:
$$A \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} B + C \underset{k_{-2}}{\overset{k_2}{\rightleftarrows}} D$$
7. Starting with the reaction scheme,

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} ES \xrightarrow{k_2} E + P$$

write the time rate of change of the molar concentrations of each of the species in the enzyme-catalyzed reaction where E is the enzyme, S is the substrate, and P is the product. Given that the total enzyme concentration, $[E]_t$, is the sum of [E] and [ES], and assuming that [ES] does not change appreciably in time, derive the expression for Michaelis-Menten kinetics (note, you may also assume that $k_{-1} >> k_2$).

8. Bacteria have evolved the ability to swim via a rotating flagellum acting like a propeller at velocities of ~10 μm/s. Assuming the bacterium is approximately spherical with radius 0.7 μm, and that it is swimming through water at 23ºC (viscosity= 0.001 kg/m s), calculate the drag force exerted by the fluid on the bacterium.
9. Our cells must often endure physical forces that cause their deformation. In some cases the forces can actually stimulate important cellular processes, such as cell growth (recall the movie of the neuron growing under tension applied via a magnetic bead). Animal cells have a Young's modulus E=~1 kPa (~6 orders of magnitude smaller than a protein!). Assuming that the cell is a cube with edge length of 10 μm, and that it is linear elastic, estimate the force required to extend the cell by 0.5 μm in the direction of applied tension.